# Integrating Spatial Motion Generation into the Engineering Curriculum

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## Abstract

**In this paper we present our effort to integrate spatial motion generation into an undergraduate mechanisms course. Our approach has focused upon incorporating finite position synthesis of spherical four bar mechanisms, whose synthesis equations closely resemble those of the planar four bar mechanism, into the curriculum. In the course students use the computer-aided design package *Sphinx*. *Sphinx* is an interactive graphics package for designing spherical mechanisms developed at U.C. Irvine and Florida Tech. Our experience has been that students using *Sphinx* gain the skills necessary to visualize three dimensional motion. Moreover, in utilizing the software students acquire the ability to design and build machines that generate spatial motions.**

## 1 Introduction

This paper discusses our efforts to integrate spatial mechanism design into an undergraduate mechanism design course. The motivation for this being that in the modern manufacturing environment we have found that engineers are being called upon to design mechanisms and machines that provide spatial motions. In the near past engineers turned to robots to provide spatial motions. However, in most applications robots have proven to be a cost ineffective solution. Hence, engineers are turning to mechanisms and linkages to generate the required spatial motions.

For six years we have worked to incorporate finite position synthesis of spherical $4R$ mechanisms into a standard course on $4R$ planar mechanism analysis and design. Spherical $4R$ mechanisms have four rigid bodies connected by revolute joints whose axes intersect at a point, in contrast to planar mechanisms which have axes that are parallel. For more information regarding spherical mechanisms see Chiang, 1988, and Duffy, 1980. The result of our efforts has been the development and student use of *Sphinx*, an interactive graphics package for designing spherical mechanisms.

In the course we use a linear algebra formulation so that, mathematically, planar and spherical finite position synthesis look the same. However, a major difference arises due to the essential three dimensionality of spherical mechanisms. Our experience has been that the leap from the $2D$ graphical constructions of planar mechanism design to $3D$ space is not difficult conceptually, however, in space the constructions can not physically be performed. Therefore the students benefit from two visualization tools we have developed: *Sphinx* and the spherical mechanism prototyping kit. In using the software, students gain the visual insight needed to grasp and comprehend the concept of spherical motion, thereby enabling them to successfully design spherical mechanisms. Experience has demonstrated that the knowledge gained by students while using *Sphinx* provides them with the ability to construct simple, inexpensive, and functional spherical $4R$ mechanisms using the spherical mechanism prototyping kit. Using the kit, students develop their intuition and manufacturing skills for spherical mechanism construction.

Spherical mechanisms generate new and interesting movements that are three dimensional by nature. Moreover, since general spatial motion can be studied as a a combination of linear and rotational motions, once spherical mechanisms have been understood, the student is well prepared for an introduction to the study of spatial geometry; especially robotics, computational geometry, and geometric modeling.

## 2 Spatial Displacements

We now present the equation for the general spatial displacement of a point in terms of the translation and rotation of its reference frame with respect to the frame's initial position, see Bottema and Roth, 1979. The coordinates of a general point x, measured in the moving frame $M$, initially coincident with a frame $F$ (which may be the fixed reference frame), which undergoes a general spatial displacement satisfy, see Fig. 1,

$$\mathbf{X} = [A]\mathbf{x} + \mathbf{d} \tag{1}$$

where, $[A]$ is the $3 \times 3$ orthonormal rotation matrix representing the orientation of frame $M$ relative to frame $F$ and $\mathbf{d} = [d_x, d_y, d_z]^T$ is the translation vector from the origin of frame $F$ to the origin of frame $M$.

For most students a general spatial motion is a difficult concept to grasp. We decompose spatial displacements into terms involving pure translations, $\mathbf{d}$, and pure rotations, $[A]$, to simplify the concept of spatial motion. In general students can visualize a spatial translation. However, our experience has been that spatial rotations pose a greater challenge to students. Since all spatial rotations can be viewed as motion on the surface of a sphere and because the links of a spherical mechanism are constrained to move on the surface of a sphere, spherical mechanism design serves as an ideal tool for teaching the visualization of spatial rotations.

## 3 Planar 4*R* Synthesis

A planar *RR* dyad is shown in Fig. 2. Let the axis of the fixed joint be specified by the vector, $\mathbf{u}$, measured in the fixed reference frame and let the moving axis be specified by, $\boldsymbol{\lambda}$, measured in the moving frame, $M$. Let us define the vector $\mathbf{v}$ as representing the moving axis $\boldsymbol{\lambda}$ as measured in the fixed frame $F$. The vectors $\mathbf{v}$ and $\boldsymbol{\lambda}$ are related by, $\mathbf{v} = [A]\boldsymbol{\lambda} + \mathbf{d}$. Because the two axes are connected by a rigid link the distance between the

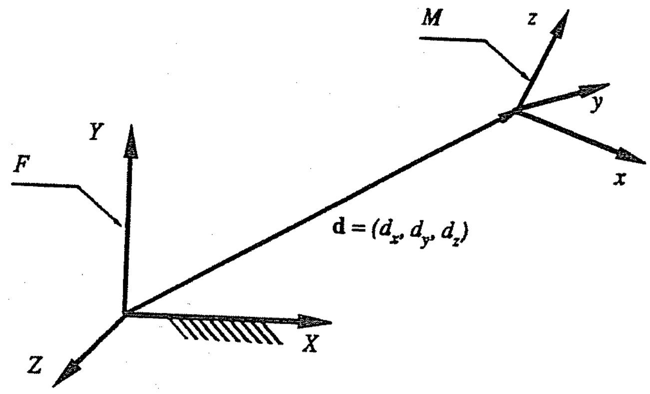


Figure 1: A General Spatial Displacement

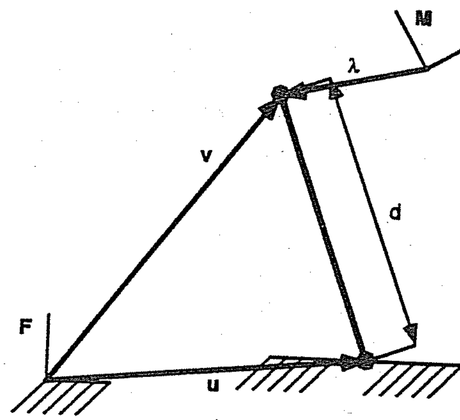


Figure 2: A Planar *RR* Dyad

two axes of the dyad remains constant. This rigid body geometric constraint is written in equation form as,

$$\begin{aligned} (\mathbf{v} - \mathbf{u}) \cdot (\mathbf{v} - \mathbf{u}) &= d^2 \\ ([A]\boldsymbol{\lambda} + \mathbf{d} - \mathbf{u}) \cdot ([A]\boldsymbol{\lambda} - \mathbf{d} - \mathbf{u}) &= d^2 \end{aligned} \tag{2}$$

The methodology for performing the dimensional synthesis of planar 4*R* mechanisms for three position rigid body guidance is as follows, see Sandor and Erdman, 1991, and Suh and Radcliffe, 1978. This algorithm is based upon synthesizing two dyads separately and then joining them with a coupler to form a planar 4*R* closed chain mechanism. First, we select the two moving pivots $\boldsymbol{\lambda}_1$ and $\boldsymbol{\lambda}_2$. Second, we write Eq. 2 for each of the desired positions, $([A], \mathbf{d})_i, i = 1, 2, 3$. Then, we subtract

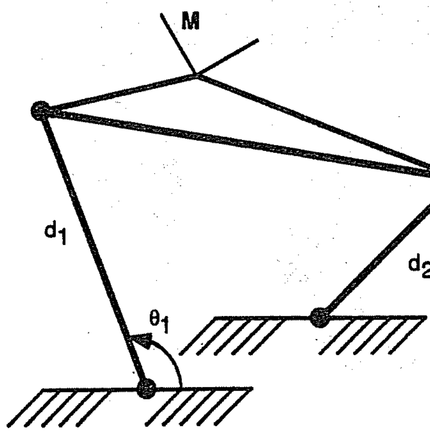


Figure 3: A Planar $4R$ Mechanism

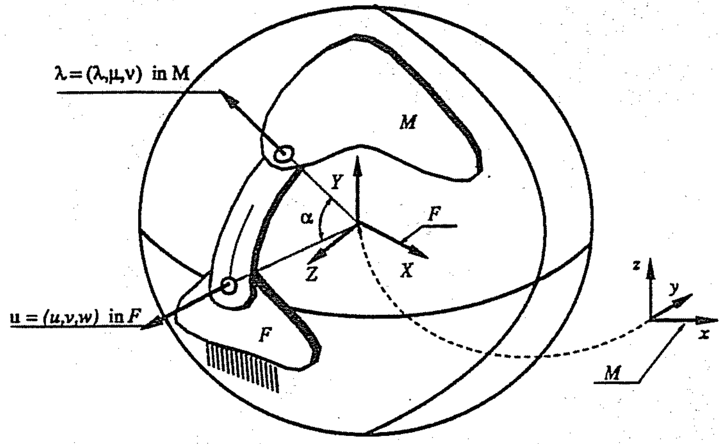


Figure 4: A Spherical $RR$ Dyad

the first equation from the remaining two to arrive at a linear system of equations,

$$[P]\mathbf{u} = \mathbf{b} \tag{3}$$

where,

$$[P] = \begin{bmatrix} 2(\mathbf{v}_2^T - \mathbf{v}_1^T) \\ 2(\mathbf{v}_3^T - \mathbf{v}_1^T) \end{bmatrix}$$

$$\mathbf{b} = \begin{bmatrix} \mathbf{v}_2^T\mathbf{v}_2 - \mathbf{v}_1^T\mathbf{v}_1 \\ \mathbf{v}_3^T\mathbf{v}_3 - \mathbf{v}_1^T\mathbf{v}_1 \end{bmatrix}$$

$\mathbf{v}_i$ are the coordinates of the moving pivot in the $i^{th}$ position, and finally $\mathbf{u}$ is the desired fixed pivot. Note that we must solve Eq. 3 for each desired moving pivot to find its corresponding fixed pivot. A complete planar $4R$ mechanism is shown in Fig. 3.

## 4 Spherical $4R$ Synthesis

A spherical $RR$ dyad is shown in Fig. 4. Let the axis of the fixed joint be specified by the vector, $\mathbf{u}$, measured in the fixed reference frame and let the moving axis be specified by, $\boldsymbol{\lambda}$, measured in the moving frame, $M$. Because the two axes are connected by a rigid link the angle between the two lines of the dyad remains constant. This rigid body geometric constraint is written in equation form as,

$$\begin{aligned} \mathbf{v}\cdot\mathbf{u} &= \cos\alpha \\ [A]\boldsymbol{\lambda}\cdot\mathbf{u} &= \cos\alpha \end{aligned} \tag{4}$$

The methodology for the dimensional synthesis of spherical $4R$ mechanisms for three position rigid body guidance follows the same procedure as outlined above for planar mechanisms. Performing the appropriate steps we arrive at,

$$[P]\mathbf{u} = \mathbf{b} \tag{5}$$

where,

$$[P] = \begin{bmatrix} \boldsymbol{\lambda}^T([A]_2 - [A]_1)^T \\ \boldsymbol{\lambda}^T([A]_3 - [A]_1)^T \\ 0 \quad 0 \quad 1 \end{bmatrix}$$

$$\mathbf{b} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

and $\mathbf{u}$ is the desired fixed pivot. Note that we must solve Eq. 5 for each desired moving pivot to find its corresponding fixed pivot. A complete spherical $4R$ mechanism is shown in Fig. 5.

## 5 $\mathcal{S}$PHINX

$\mathcal{S}$PHINX is a computer graphics based interactive program for designing spherical $4R$ mechanisms. For further discussion of $\mathcal{S}$PHINX, see Larochelle, 1994 and Larochelle et al, 1993. The result of the design is a one degree of freedom mechanism which guides a body through finitely separated orientations in space. In $\mathcal{S}$PHINX the user specifies orientations, using longitude, latitude, and roll angles, which are

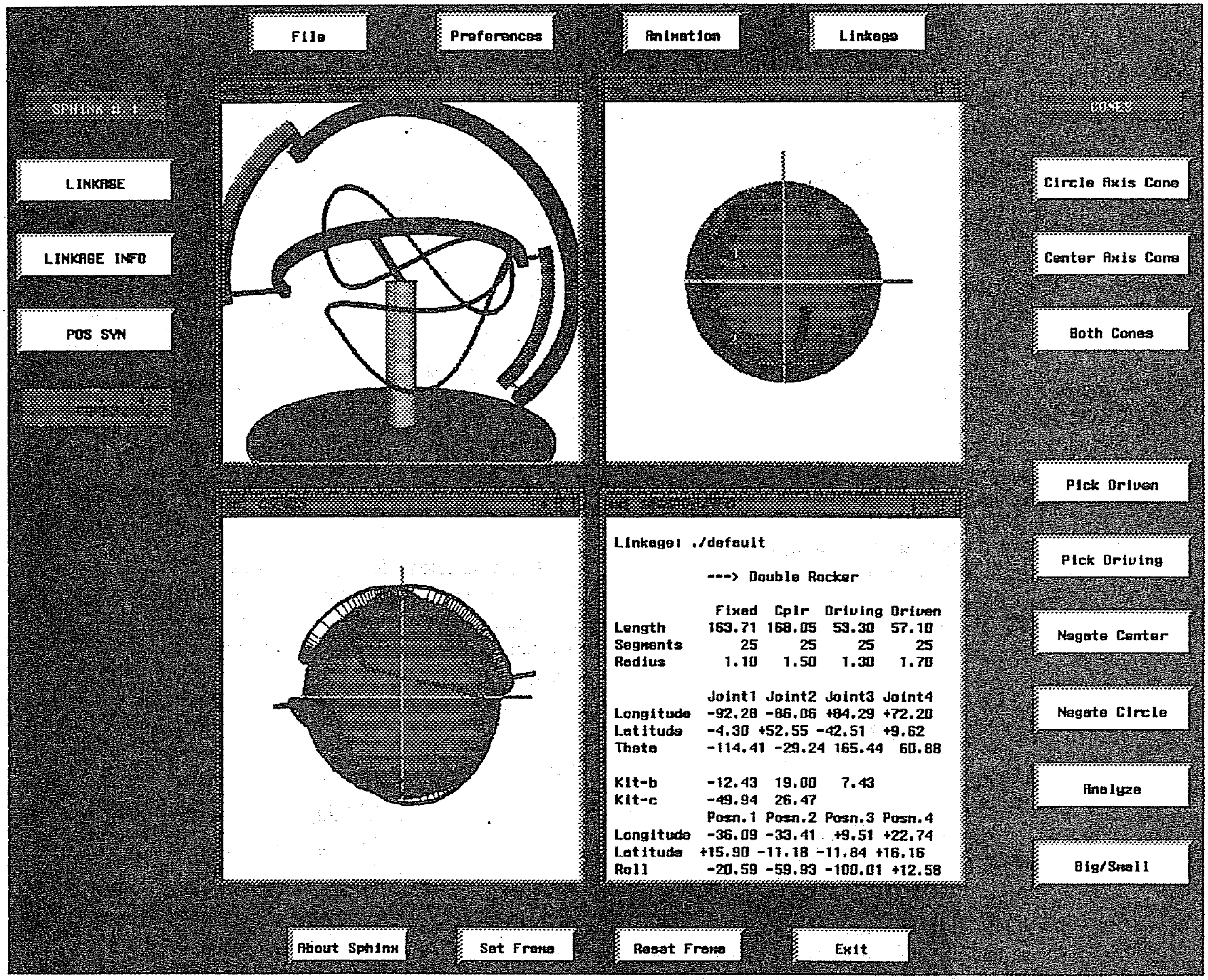


Figure 6: *S*PHINX

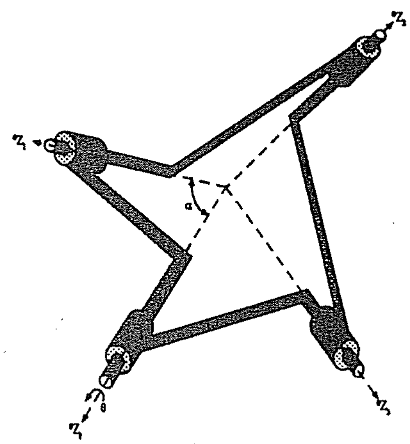

Figure 5: A Spherical $4R$ Mechanism

displayed as positions on the surface of a sphere. This display is generated by translating the reference frame of the orientation from the origin of the sphere along its $z$ axis a unit distance so that the reference frame appears on the surface of a unit sphere, see Fig. 6. Our experience has been that this technique enhances the student's ability to visualize spatial orientations. If the translation were not done all of the orientations would have coinciding origins at the center of the sphere thereby making visualization difficult.

The major difference between planar and spherical finite position synthesis is the essential three dimensionality of spherical mechanisms. We have found that static visualization is insufficient and that in order to successfully design spherical mechanisms the designer requires the ability to interactively manipulate the sphere and view it in an arbitrary orientation. SPHINX uses the graphics capabilities of a Silicon Graphics *Indigo*$^2$ XL, to provide the three dimensional interactive environment needed to design spherical $4R$ mechanisms.

# 6 The Prototyping Kit

We have designed, developed, and manufactured a spherical mechanism prototyping kit(patent pending) which can be used independently, or in conjunction with SPHINX. The purpose of the kit is to aid students in the construction of an initial prototype of their design. By physically constructing their designs students gain an appreciation of machines and mechanisms that can provided spatial and spherical motions. Moreover, they develop basic manufacturing skills for spherical mechanism construction.

# 7 Conclusion

Our experience in integrating spherical mechanism design in the engineering curriculum with SPHINX and the spherical mechanism prototyping kit has had the following results: (1) provided a methodology for students to understand, and develop an intuition for, spatial motion. (2) sparked interest and enthusiasm in students in that they are designing and building exciting three dimensional machines. (3) resulted in students that are well prepared for an introduction to more involved studies of spatial geometry; including robotics, computational geometry, and geometric modeling.

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Prof. Larochelle received his Ph.D. in mechanical engineering from the University of California, Irvine for research concerning the design and analysis of spatial mechanisms and cooperating robots in June of 1994. Upon completion of his doctorate he served as a post doctoral researcher at the University of California, Irvine where he developed new computer-aided mechanism design techniques. In winter of 1995 he joined the mechanical engineering program at the Florida Institute of Technology. His research interests include theoretical kinematics, computer-aided design of spatial and spherical mechanisms, static and dynamic analyses of closed chains and cooperating robots, and controls. Prof. Larochelle is a member of ASME, IEEE, and ASEE.