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SPADES: SOFTWARE FOR SYNTHESIZING SPATIAL 4C MECHANISMS

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ABSTRACT

In this paper we present SPADES, an interactive graphics based software package for ***SPA***tial mechanism ***DES***ign. The program provides a platform for the synthesis of a mechanism that guides a body through either three or four prescribed positions in space.

The purpose of this work is to assemble the current spatial 4*C* synthesis theory into a software package that is useful for spatial mechanism design and research.

INTRODUCTION

The synthesis of planar mechanisms is inherently a two dimensional problem. Therefore, the design techniques are well suited to a drafting table, blackboard, etc. This is not true of spatial mechanisms. The inherent three dimensionality of these mechanisms makes such two dimensional graphical constructions difficult. For these mechanisms it is useful for the designer to be able to visualize the entire problem in its three dimensions. Modern computer workstations provide the high speed graphics capabilities which make possible real-time visualization of spatial mechanisms. SPADES uses the three dimensional graphics capabilities of a Silicon Graphics *Indigo*^{2} desktop workstation to provide the interactive environment needed to design spatial 4*C* mechanisms. Previous efforts which utilized three dimensional graphics in computer-aided mechanism design are summarized in Erdman (1995) and Erdman (1993) and are presented in Rubel and Kaufman (1977), Barris, Kota, Riley, and Erdman (1988), Thatch, and Myklebust (1988), and Larochelle et al (1993).

SPADES is a computer graphics based interactive program for designing spatial mechanisms formed by a closed chain consisting of four cylindric(*C*) joints. Cylindric joints allow both translation along and rotation about an axis, hence, they are two degree of freedom joints. The result of rigidly connecting four *C* joints is a two degree of freedom spatial closed chain mechanism, referred to as a spatial 4*C* mechanism. SPADES facilitates the synthesis of spatial 4*C* mechanisms to guide a body through three or four finitely separated positions in space.

The theory for the design of spatial mechanisms for four position rigid body guidance is analogous to that for planar mechanisms. In the planar case the designer specifies four positions in the plane and computes the set of points in the moving body which have four positions on a circle, see Hartenberg and Denavit (1964) and Burmester (1888). These points are the moving pivots of planar *RR* dyads compatible with the four positions and they form a circular cubic curve called the circle point curve. The points that are the centers of these circles are the corresponding fixed pivots of the planar *RR* dyads and they form a cubic curve called the center point curve. In the case of spatial 4*C* synthesis the designer specifies four positions in space. The set of lines in the moving body which maintain a constant distance(i.e. normal distance and twist) from a fixed line form a congruence of lines called the moving line congruence. The set of corresponding fixed lines of the *CC* dyads form the fixed line congruence, see Larochelle (1995) and Bottema and Roth (1979) for further discussions about these line congruences and their properties. The major difference between planar and spatial finite position synthesis is the essential three dimensionality of spatial mechanisms. While synthesis curves for planar mechanisms may be sketched or plotted in two dimensions the synthesis congruences for spatial mechanisms and the mechanisms themselves must be viewed in their full three dimensional form. The designer requires the ability to manipulate the synthesis congruences and view them in an arbitrary orientation in order to gain an understanding of the available choices of fixed and moving axes. Furthermore, once the linkage has been specified the evaluation of its motion also requires the ability to view the linkage in a three dimensional environment.

The remainder of the paper proceeds as follows. First, we review the underlying synthesis theory utilized by *S*PADES to perform synthesis for three and four position rigid body guidance. This is followed by a summary of the kinematic analysis tools employed by *S*PADES and a discussion regarding the organizational structure of the software. Finally, we conclude with a design case study which illustrates the utility of *S*PADES as a research and design tool and discuss the planned future development of *S*PADES.

THREE POSITION SYNTHESIS

The methodology utilized for performing the dimensional synthesis of spatial 4*C* mechanisms for three position rigid body guidance involves synthesizing two *CC* dyads separately and then joining them with a coupler to form a complete closed chain mechanism. This algorithm is based upon the works of Larochelle (1994), Suh and Radcliffe (1978), and Tsai and Roth (1973).

We consider one *CC* dyad of the spatial 4*C* mechanism as shown in Fig. 1. Let the axis of the fixed joint be specified by the dual vector $\hat{\mathbf{u}}$ measured in the fixed reference frame and let the moving axis be specified by $\hat{\lambda}$ measured in the moving frame *M*, see Bottema and Roth (1979) for further discussion of dual vectors.

In a *CC* dyad there is a link which connects the moving line and the fixed line. This link is assumed to be rigid and therefore the twist and normal distance between the fixed and moving lines of the dyad remain constant. The constant twist condition may be expressed as,

$$\mathbf{u}\cdot[A]\lambda - \cos\alpha = 0 \quad (1)$$

and the constant moment condition as,

$$\mathbf{u}\cdot([A]\lambda^0 + [D][A]\lambda) + \mathbf{u}^0\cdot[A]\lambda + a\sin\alpha = 0 \quad (2)$$

where $[A]$ is the 3×3 rotation matrix which describes the orientation of the moving body with respect to the fixed frame and $[D]$ is the 3×3 skew-symmetric matrix from the translation vector $\mathbf{d}$ which locates the origin of the moving frame. Note that Eq. 1 and Eq. 2 are the implicit constraint equations for a spatial *CC* dyad.

For three position synthesis we first select the moving axis $\hat{\lambda}$. Second, we write Eq. 1 for each of the desired positions, $([A],\mathbf{d})_i, i = 1,2,3$. Finally, we subtract the first equation from the remaining two to arrive at a linear system of equations,

$$[P]\mathbf{u} = \mathbf{b} \quad (3)$$

where,

$$[P] = \begin{bmatrix} (\mathbf{l}_2 - \mathbf{l}_1)^T \\ (\mathbf{l}_3 - \mathbf{l}_1)^T \\ 0\ 0\quad 1 \end{bmatrix}$$

$$\mathbf{b} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$\mathbf{l}_i$ are the coordinates of the moving pivot in the i^{th} position, and $\mathbf{u}$ is a vector in the direction of the desired fixed pivot. We now solve the system of equations and normalize the solution to obtain the vector $\mathbf{u}$. Next, write Eq. 2 for each of the desired positions, $([A],\mathbf{d})_i, i = 1,2,3$. Again, we subtract the first equation from the remaining two to arrive at a linear system of equations,

$$[P]\mathbf{u}^0 = \mathbf{b} \quad (4)$$

where,

$$[P] = \begin{bmatrix} (\mathbf{l}_2 - \mathbf{l}_1)^T \\ (\mathbf{l}_3 - \mathbf{l}_1)^T \\ \mathbf{u}^T \end{bmatrix}$$

$$\mathbf{b} = \begin{pmatrix} -(\mathbf{l}_2^0 - \mathbf{l}_1^0)\cdot\mathbf{u} \\ -(\mathbf{l}_3^0 - \mathbf{l}_1^0)\cdot\mathbf{u} \\ 0 \end{pmatrix}$$

The desired fixed axis is,

$$\hat{\mathbf{u}} = \begin{pmatrix} \mathbf{u} \\ \mathbf{u}^0 \end{pmatrix}. \quad (5)$$

Note that the fixed axis $\hat{\mathbf{u}}$ which corresponds to the desired $\hat{\lambda}$ is unique and that we must perform this procedure for both desired moving axes in order to complete the 4*C* mechanism synthesis.

FOUR POSITION SYNTHESIS

The design congruences for four position synthesis of spatial $4C$ closed chain mechanisms are called the fixed line and moving line congruences. They are used in the same way as the planar linkage design curves; the center point and circle point curves. The fixed line congruence is the set of lines that will serve as fixed axes and the moving line congruence is the set of corresponding moving axes. The numerical procedure we use to generate these congruences is presented in detail in Larochelle (1995) and we briefly summarize the process here.

The generalization of Burmester's planar four position theory to four spatial displacements leads us to consider the complementary screw quadrilateral $S_{12}S_{23}S_{34}S_{14}$, where S_{12}, S_{13}, S_{14}, S_{23}, S_{24}, and S_{34} are the six relative screw axes associated with the four prescribed spatial positions of a moving body, see Fig. 2 and Roth (1967a,1967b) and Bottema and Roth (1979). We use the lines which define the complementary screw quadrilateral to define a spatial $4C$ mechanism and identify the quadrilateral as the *home* configuration of the parameterizing $4C$ mechanism. The screw axis of the displacement of the coupler of the parameterizing $4C$ mechanism from its home configuration to any other valid assembly is a fixed axis compatible with the given four spatial positions and hence is a line of the fixed line congruence. Murray and McCarthy (1994) show that solving the spatial triangle associated with the two lines which define the input crank in its home configuration, Eq. 6 and Eq. 7, results in the screw axis of the displacement of the coupler of the parameterizing $4C$ mechanism. The dual vector equation of this spatial triangle(shown in Fig. 3) may be written as,

$$\sin\frac{\hat{\beta}}{2}\hat{\mathbf{g}} = \sin\frac{\Delta\hat{\theta}}{2}\cos\frac{\Delta\hat{\phi}}{2}S_{12} + \cos\frac{\Delta\hat{\theta}}{2}\sin\frac{\Delta\hat{\phi}}{2}S_{23} + \sin\frac{\Delta\hat{\theta}}{2}\sin\frac{\Delta\hat{\phi}}{2}S_{12}\times S_{23} \tag{6}$$

$$\cos\frac{\hat{\beta}}{2} = \cos\frac{\Delta\hat{\theta}}{2}\cos\frac{\Delta\hat{\phi}}{2} + \sin\frac{\Delta\hat{\theta}}{2}\sin\frac{\Delta\hat{\phi}}{2}S_{12}\cdot S_{23} \tag{7}$$

where $\Delta\hat{\theta} = \hat{\theta} - \hat{\theta}_0$, $\Delta\hat{\phi} = \hat{\phi} - \hat{\phi}_0$, $\hat{\theta}_0$ is the relative dual angle between the the input crank and the fixed link in the home configuration, and $\hat{\phi}_0$ is the relative dual angle between the coupler and the input crank in the home configuration. By solving the spatial triangle we obtain a fixed axis compatible with the given four spatial positions which is parameterized by the input angle of the parameterizing $4C$ linkage.

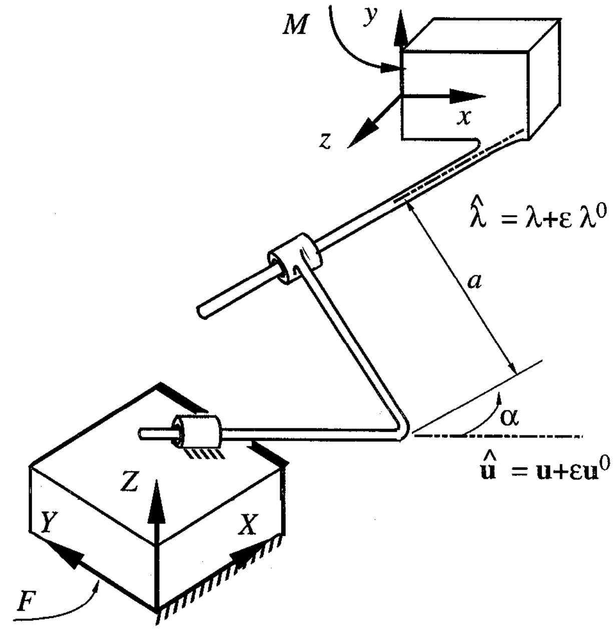

Figure 1. A SPATIAL CC DYAD

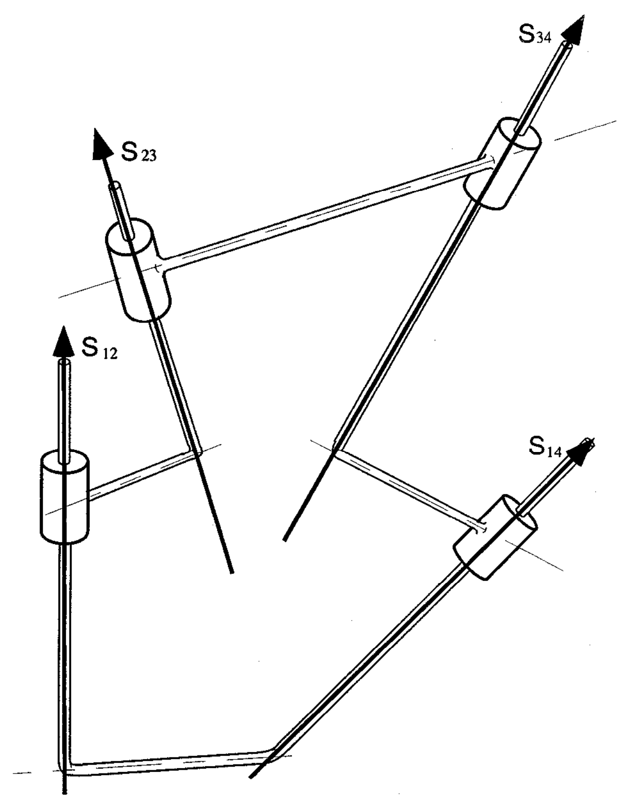

Figure 2. THE PARAMETERIZING SPATIAL $4C$ MECHANISM

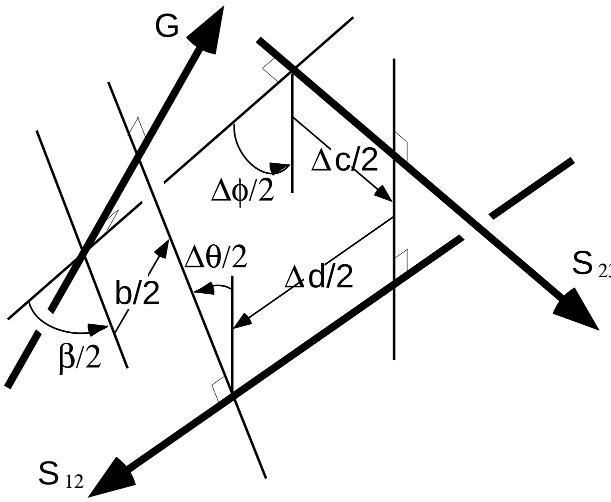

Figure 3. THE SPATIAL TRIANGLE

Bottema and Roth (1979) and Roth (1967c) have shown that the direction of each line $\hat{\mathbf{g}}$ determines a plane and that all of the lines in that plane that are parallel to $\hat{\mathbf{g}}$ are members of the fixed line congruence. To obtain another line in the same plane as $\hat{\mathbf{g}}$ we maintain θ and vary our choice of d, where d is the translation of the input crank of the parameterizing linkage along S_{12}, ($\hat{\theta} = \theta + \varepsilon d$), and solve Eq. 6 and Eq. 7. These two lines then define a plane of the congruence and any line in this plane which is parallel to them is also a member of the fixed line congruence. The moving line congruence is obtained by inverting the relationship between the fixed and moving coordinate frames and proceeding in a manner analogous to the generation of the fixed line congruence.

There is a one-to-one correspondence between lines of the fixed line congruence and lines of the moving line congruence. That is to say, selecting a line from the fixed line congruence as the fixed axis of a *CC* dyad uniquely determines the moving axis, and vice verse, see Roth (1967c).

KINEMATIC ANALYSIS

Here we introduce the notation for the link lengths and joint angles. We then present the equations which are utilized to perform a kinematic analysis of the spatial 4*C* mechanism. These equations are derived at by utilizing the matrix method of Hartenberg and Denavit (1964). Finally, we present the linkage classification method employed by *S*PADES.

Position Analysis

The link lengths and joint angles are as defined in Tbl. 1. The coupler angle φ as a function of the input angle θ is,

$$\phi(\theta) = \arctan(\frac{B}{A}) \pm \arccos(\frac{C}{\sqrt{A^2+B^2}}) \tag{8}$$

where

$$\begin{aligned} A =\ & \sin(\eta)\sin(\gamma)\cos(\alpha)\cos(\theta) - \\ & \sin(\alpha)\sin(\eta)\cos(\gamma) \\ B =\ & -\sin(\eta)\sin(\gamma)\sin(\theta) \\ C =\ & \cos(\beta) - \cos(\eta)\sin(\alpha)\sin(\gamma)\cos(\theta) - \\ & \cos(\alpha)\cos(\eta)\cos(\gamma). \end{aligned}$$

The output angle ψ as a function of the input angle θ and the coupler angle φ is,

$$\psi(\theta, \phi) = \arctan(\frac{B}{A}) \tag{9}$$

where,

$$\begin{aligned} A =\ & \frac{1}{-\sin(\beta)}\{\cos(\eta)(\cos(\alpha)\sin(\gamma) - \\ & \cos(\gamma)\cos(\theta)\sin(\alpha)) - \\ & \sin(\eta)\cos(\phi)(\cos(\alpha)\cos(\gamma)\cos(\theta) + \\ & \sin(\alpha)\sin(\gamma)) + \\ & \sin(\eta)\cos(\gamma)\sin(\phi)\sin(\theta)\} \\ B =\ & \frac{1}{\sin(\beta)}\{\cos(\eta)\sin(\alpha)\sin(\theta) + \\ & \sin(\eta)\cos(\theta)\sin(\phi) + \\ & \sin(\eta)\cos(\alpha)\cos(\phi)\sin(\theta)\}. \end{aligned}$$

The output coupler angle δ, i.e. the angle between the coupler and the driven crank is,

$$\delta(\theta, \psi) = \arctan(\frac{B}{A}) \tag{10}$$

where,

$$\begin{aligned} A =\ & \frac{1}{\sin(\eta)}\{\cos(\alpha)(\cos(\gamma)\sin(\beta) + \\ & \cos(\beta)\sin(\gamma)\cos(\psi)) - \end{aligned}$$

Table 1. LINK LENGTH AND JOINT ANGLE NOTATION

Link	Dual Angle	Twist	Length
Driving/Input	$\hat{\alpha}$	α	a
Coupler	$\hat{\eta}$	η	h
Driven/Output	$\hat{\beta}$	β	b
Fixed	$\hat{\gamma}$	γ	g
Joint	**Dual Angle**	**Rotation**	**Translation**
Driving Fixed	$\hat{\theta}$	θ	d_1
Driving Moving	$\hat{\phi}$	ϕ	c_1
Driven Moving	$\hat{\delta}$	δ	c_2
Driven Fixed	$\hat{\psi}$	ψ	d_2

$$\begin{aligned}
&\sin(\alpha)\cos(\theta)(\cos(\beta)\cos(\gamma)\cos(\psi) - \\
&\sin(\beta)\sin(\gamma)) - \\
&\sin(\alpha)\cos(\beta)\sin(\theta)\sin(\psi)\} \\
B = &\frac{1}{-\sin(\eta)}\{\cos(\alpha)\sin(\gamma)\sin(\psi) + \\
&\sin(\alpha)\sin(\theta)\cos(\psi) - \\
&\sin(\alpha)\cos(\gamma)\cos(\theta)\sin(\psi)\}.
\end{aligned}$$

The driving coupler translation c_1 (i.e. the translation along the driving moving axis) is given by,

$$c_1(\theta,\psi,\delta,d_1) = \frac{A}{B} \tag{11}$$

where,

$$\begin{aligned}
A = &\, d_1\sin(\gamma)\sin(\psi) + a\cos(\theta)\cos(\psi) + \\
&a\cos(\gamma)\sin(\theta)\sin(\psi) + h\cos(\delta) - \\
&-b - g\cos(\psi) \\
B = &\sin(\eta)\sin(\delta).
\end{aligned}$$

The driven coupler translation c_2 (i.e. the translation along the driven moving axis) is given by,

$$c_2(\theta,\phi,\psi,c_1) = \frac{A}{B} \tag{12}$$

where,

$$\begin{aligned}
A = &\, h\cos(\phi)\cos(\theta) + c_1\sin(\alpha)\sin(\theta) + \\
&a\cos(\theta) - h\cos(\alpha)\sin(\phi)\sin(\theta) - \\
&g - b\cos(\psi) \\
B = &\sin(\beta)\sin(\psi).
\end{aligned}$$

Finally, the translation along the driven fixed line d_2 is given by,

$$d_2(\theta,\phi,\psi,c_1,c_2) = \frac{A}{B} \tag{13}$$

where,

$$\begin{aligned}
A = &\, h\cos(\phi)\sin(\theta) - c_1\cos(\theta)\sin(\alpha) + \\
&a\sin(\theta) + h\cos(\alpha)\cos(\theta)\sin(\phi) - \\
&b\cos(\gamma)\sin(\psi) + c_2\cos(\beta)\sin(\gamma) + \\
&c_2\cos(\gamma)\cos(\psi)\sin(\beta) \\
B = &-\sin(\gamma).
\end{aligned}$$

Mechanism Classification

Associated with each spatial 4*C* mechanism is a spherical image. The spherical image is a spherical four-bar mechanism with link lengths equal to the angular twist of the links of the 4*C* mechanism, see Duffy (1980). SPADES classifies spatial 4*C* mechanisms according to the linkage type of the corresponding spherical image. The linkage type of the spherical image is computed using the method presented in Murray and McCarthy (1995).

SOFTWARE STRUCTURE

SPADES is structured in such a manner as to provide an interactive platform from which the designer creates linkages in order

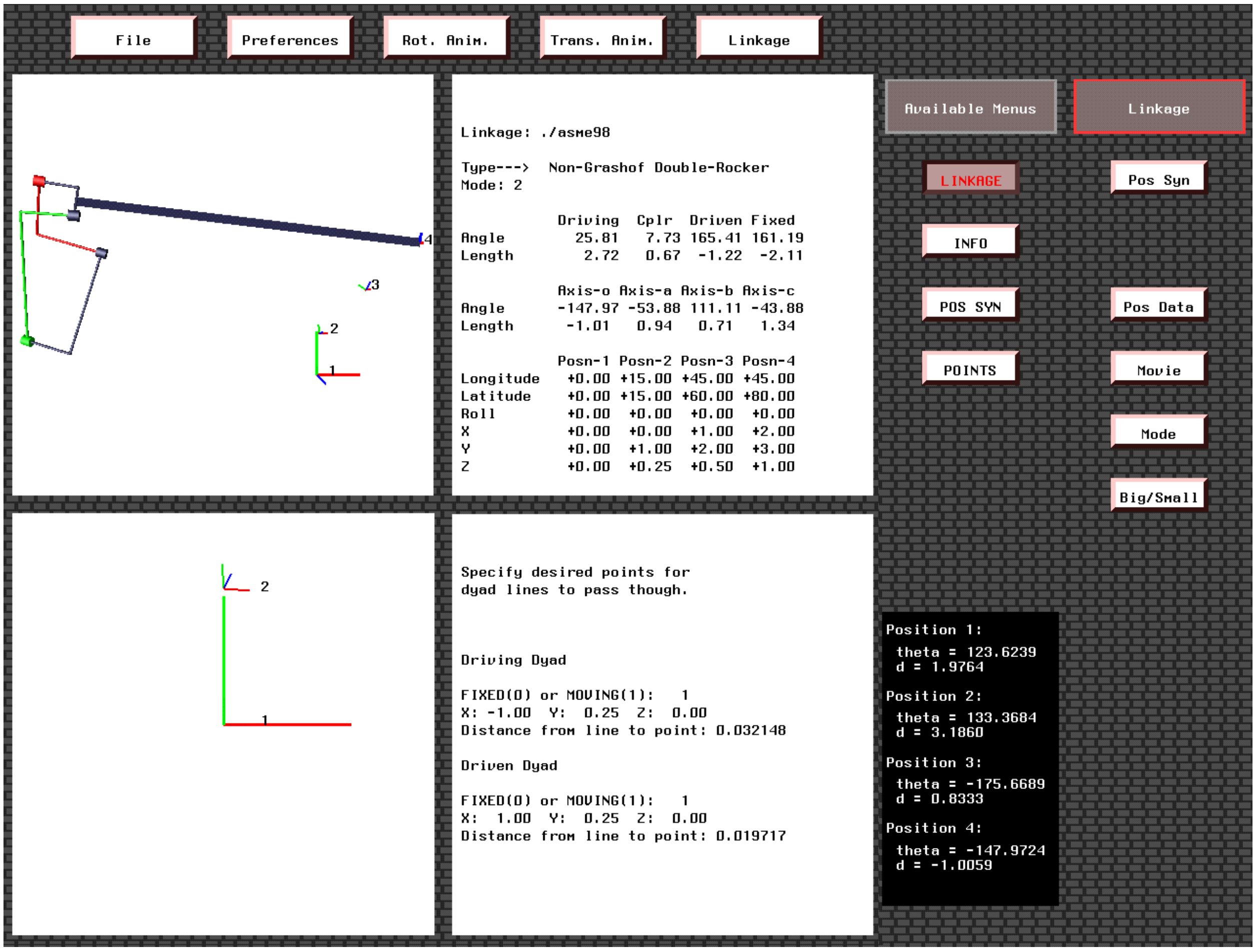

Figure 4. THE SPADES DESIGN ENVIRONMENT

to satisfy given design specifications. The program is organized such that each phase of the design process is carried out in a separate graphics window. Corresponding to each window is a button listed on the right hand side of the screen in the *available menus* column labeled with the name of the window. To select a window to work in use the mouse to push the appropriate button in the *available menus* column. The menu associated with that graphics window is then displayed on the far right hand side of the screen, see Fig. 4.

At start-up there are two windows displayed; the *linkage* window and the *linkage info* window. The menu associated with the *linkage* window is displayed on the far right side of the screen. There is no menu associated with the *linkage info* window. To begin the design process buttons in the right menu are pressed. These buttons include *Pos Syn*, *Pos Data*, *Movie*, and *Mode*. As the buttons are pressed the required windows and their associated buttons and menus are created.

Position Synthesis

To design a linkage to guide a body through three or four positions in space press the *pos syn* button in the *linkage* menu. The *pos syn* window will be created as well as the *pos syn* menu. Four positions are represented as coordinate frames in space. The position frames have their axes color coded; $x = red$, $y = green$, and $z = blue$. The orientation of the position frames is specified by their longitude, latitude, and roll angles, see Larochelle and McCarthy (1995). Position orientations are specified by graphically moving a frame (select the desired position number in the *pos syn* menu) to the desired orientation or by typing the desired longitude, latitude, and roll angles directly into the *linkage info* window. The location of the origin of the position is specified numerically in the *linkage info* window.

Once the four positions are specified the corresponding design congruences are generated by pressing either the *congos*, *guide map*, or *points* button in the *pos syn* menu. Alternatively, three position synthesis is accomplished by pressing the *3 pos syn* button.

Design Congruences. After the four precision positions are specified and the congruences created, the selection of the cranks of the mechanism is accomplished in the congos, guide map, or points window. By choosing the appropriate button in the *congos* menu both the driven crank and the driving crank are selected from the congruences. Picking a crank involves selecting a desired line from the design congruences. First, press either the *pk drving* or *pk drven* buttons in the *congos* menu. Then, place the mouse pointer on the desired line in the *congos* window where planes of the moving line congruence are yellow and planes of the fixed line congruence are red. Press the left or right mouse button to select a line. Both lines of the crank will then be displayed in the *congos* window. The driving crank lines are green while the driven crank lines are red. Note that the same color coding of the lines is used in the *linkage* window. After both cranks have been chosen the *analyze* button in the *congos* menu is used to create the corresponding linkage which is then displayed in the linkage window. Note that in the congos window each plane of the congruence is selected by a generator line. The ability to select a line from a plane other than its generator is enabled by selecting *use generators* in the preferences menu. This will allow you to select a line from a plane that passes through, or nearest, a desired point.

Guide Map. After the four precision positions are specified the selection of the cranks of the mechanism can alternatively be accomplished in the guide map window. By choosing the appropriate button in the *pos syn* menu a guide map is generated. The guide map is a two-dimensional visual representation of all of the mechanisms generated by the congruences. In the guide map all possible selections of lines from the congruences are coded according to mechanism type. Pressing *col key* in the *guide map* menu will generate a mechanism type color legend. By selecting a point on the guide map with the mouse both the driven crank and the driving crank are determined and the mechanism is analyzed and displayed in the *linkage* window. The note above regarding *use generators* in the preferences menu also applies to the guide map.

Points. After the four precision positions are specified the selection of the cranks of the mechanism can also be accomplished in the points window. By choosing the appropriate button in the *pos syn* menu the points window is generated. In this window you can select a mechanism from the congruences by specifying a point that you want an axis of the mechanism to pass through. Note that the distance from the selected line of the congruence nearest the desired point is displayed and that the linkage is automatically analyzed and displayed in the linkage window.

Three Position Synthesis. After the precision positions are specified, synthesis of a mechanism that guides a moving body through the first three of the four positions can be accomplished in the 3 pos syn window. By choosing the appropriate button in the *pos syn* menu the 3 pos syn window is generated. In this window you can specify either the fixed or moving line of each crank and the corresponding moving or fixed line will be computed and the linkage will automatically be analyzed and displayed in the linkage window.

Motion Simulation

Once synthesized, the planned motion of the 4*C* mechanism can be viewed by pressing the *movie* button in the *linkage* menu. *S*PADES will plan a path for the mechanism and will actuate both degrees of freedom such that the moving body will pass through the four, or three, desired positions. The speed of the movie can be altered by selecting *adj. movie speed* in the preferences menu. Press the *movie* button again to terminate the movie function and return to normal *S*PADES operation. Note that pressing *pos data* in the linkage menu will display the driving crank rotation angle and translation distance in each of the four positions.

Pull-Down Menus

At the top of the main *S*PADES display are standard windows pull-down menus which open and close files that contain saved designs as well as three additional menus; *Preferences*, *Rot. Anim.*, and, *Trans. Anim.* The *Preferences* menu presents various options which are made available to the user throughout the design process to enhance the capabilities of *S*PADES:

Linkage Colors- Toggles between a steel grey mechanism and one which has a green driving crank and a red driven crank.
Show Axes- Toggles display of the fixed frame axes in the linkage window.
Show Pos-n-Linkage- Toggles display of the four desired positions in the linkage window.
Use Generators- Toggles display of a dialogue window to prompt the user to select a specific line in a plane of the congruence rather than use the line which was used to generate that plane.
Adj. Movie Speed- Toggles display of a dialogue window to prompt the user for a new movie display speed.

The *Rot. Anim.* and *Trans. Anim.* menus present options for manually actuating the two degrees of freedom at the driving fixed axis. The video cassette-player type selections available for each degree of freedom are: $Play$, $FastForward$, $ReversePlay$, $Slow - Motion$, $Super - Slow$, and $Stop$.

Table 2. THE 4 PRESCRIBED POSITIONS

Pos.	X	Y	Z	Long.	Lat.	Roll	θ	d_1
1	0.00	0.00	0.00	00.0	00.0	00.0	123.6	1.976
2	0.00	1.00	0.25	15.0	15.0	00.0	133.4	3.186
3	1.00	2.00	0.50	45.0	60.0	00.0	−175.7	0.833
4	2.00	3.00	1.0	45.0	80.0	00.0	−150.0	−1.006

DESIGN CASE STUDY

In this section we present an example of the design of a spatial 4*C* mechanism for four position rigid body guidance. The goal is to move a pallet off of a flexible assembly line into a convenient position to perform an assembly operation on the underside of the pallet and then to return the pallet to the assembly line. A moving coordinate frame was assigned to the pallet and the 4*C* mechanism is to be attached to the pallet at the points; $[-1\ 0.25\ 0]^T$ and $[1\ 0.25\ 0]^T$. It is at these two points that holes are to be drilled in the pallet. These holes will serve as journal bearings for the moving *C* joints of the 4*C* mechanism. This application was suggested by Mark Senti and his associates at GSMA Systems Inc, Melbourne, FL and was first addressed in Larochelle (1995). The four desired positions are listed in Tbl. 2. The fixed and moving congruences associated with this set of positions are shown in Fig. 5. From the computed congruences we seek a 4*C* mechanism with a driving crank which has a moving line that passes through the point $\mathbf{p}_{dvg} = [-1\ 0.25\ 0]^T$ and a driven crank with a moving line that passes through the point $\mathbf{p}_{dvn} = [1\ 0.25\ 0]^T$; both points are given with respect to the moving frame. The synthesis will be accomplished by utilizing the the *points* synthesis feature of SPADES which was previously discussed, see Fig. 4 for a snapshot of the *points* feature in use.

For each plane of the moving congruence the distance *h* from the plane to the point $\mathbf{p}_{dvg}$ was computed. The line of the moving congruence nearest the desired point $\mathbf{p}_{dvg}$ was found to be at a distance of $h_{min} = 0.032148$. We obtain the driven dyad in an analogous manner. The line of the moving congruence nearest the point $\mathbf{p}_{dvn}$ was found to be at a distance of $h_{min} = 0.019717$. SPADES automatically computes the fixed axes which correspond to these moving axes and displays the mechanism and the pertinent data in the design environment. Finally, we show the resulting non-Grashof spatial 4*C* mechanism in Fig. 6 and list its link lengths in Tbl. 3. Moreover, the dual input dual for the mechanism with the moving body in each of the four prescribed positions are found in Tbl. 2. In Fig. 6 the mechanism is shown with: a green driving crank, a red driven crank, and the moving body in position 4 and the lines which it to the coupler.

Table 3. THE DESIRED 4*C* MECHANISM

Link	Length (deg, distance)
DRIVING	(25.81, 2.718)
COUPLER	(7.73, 0.668)
DRIVEN	(165.41, −1.219)
FIXED	(161.19, −2.112)

FUTURE DEVELOPMENT

SPADES is written is the *C* programming language and utilizes the *GL* graphics library which is available on Silicon Graphics computers. In the near future we plan to port SPADES to the new *OpenGL* graphics library. This will allow SPADES to be compiled and run on any platform which has a *C* compiler and the *OpenGL* libraries.

Future synthesis tools planned for SPADES include a path planning module to plan the coordinated movement of the two degrees of freedom to guide the moving body in some optimal fashion. Possible optimization objective functions may include: minimizing the translation required along the driving fixed line, minimizing the number of required algebraic sign changes for the driving force and driving torque, minimizing the maximum required forces and torques, minimizing the required power, etc. Moreover, we plan to build upon the work of Ling and Hu (1997) and develop a module for sizing the links and locating their connections to avoid collisions during actuation of the mechanism.

CONCLUSIONS

In this article we have presented SPADES, an interactive software package for designing spatial 4*C* closed chains. Incorporated into SPADES are modules for performing synthesis for three and four position rigid body guidance. Moreover, SPADES provides the designers with three dimensional motion planning and animation of the mechanism.

It is our hope that this research and design tool will facilitate the design and manufacture of mechanisms to solve spatial motion problems. The source code for SPADES and the SPADES *User's Guide* are available, free of charge, from the author.

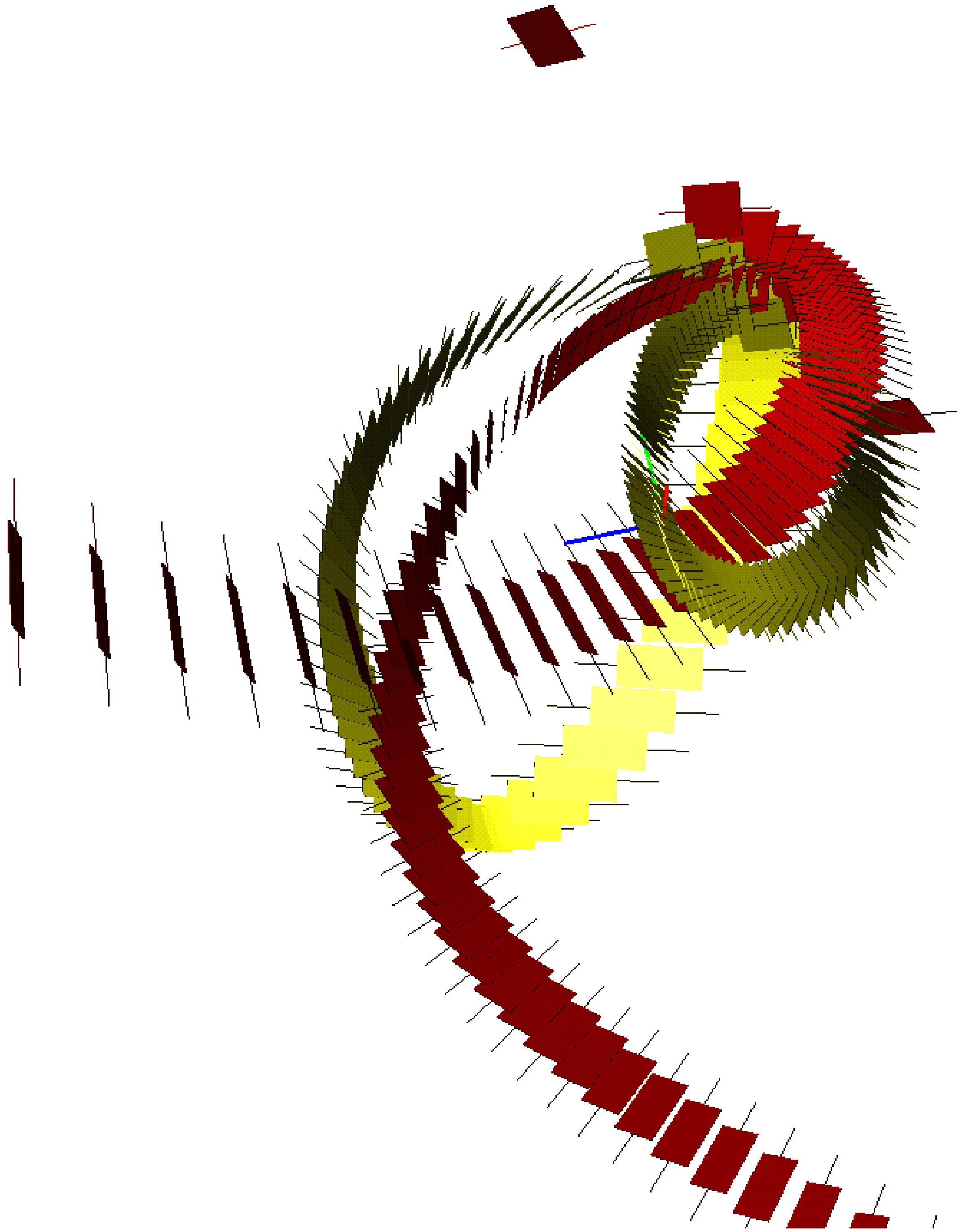

Figure 5. THE FIXED AND MOVING CONGRUENCES

ACKNOWLEDGMENTS

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REFERENCES

A. G. Erdman. Computer-aided mechanism design: Now and the future. *ASME J. of Mechanical Design*, 117:93–117, June 1995.

A. Erdman. *Modern Kinematics: Developments in the last 40*

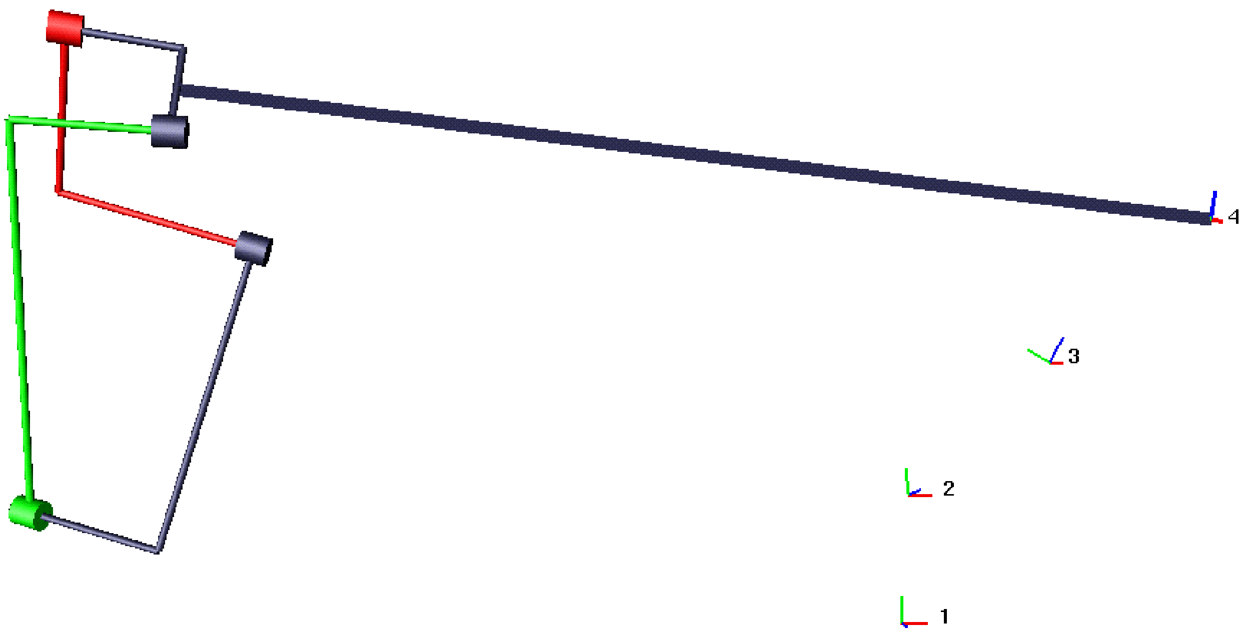

Figure 6. THE 4C MECHANISM

Years. Wiley and Sons, New York, New York, 1993.
A. J. Rubel and R. E. Kaufman. Kinsyn iii: A new human-engineered system for interactive computer-aided design of planar linkages. *ASME J. of Engr. for Industry*, 99(2):440–448, May 1977.
Wesley C. Barris, Sridhar Kota, Donald R. Riley, and Arthur G. Erdman. Mechanism synthesis using the workstation environment. *IEEE Trans. Computer Graphics and Applications*, 8(2):39–50, March 1988.
Brian R. Thatch and Arvid Myklebust. A phigs-based graphics input interface for spatial-mechanism design. *IEEE Trans. Computer Graphics and Applications*, 8(2):26–38, March 1988.
P. M. Larochelle, J. R. Dooley, A.P. Murray, and J. M. McCarthy. Sphinx: Software for synthesizing spherical 4r mechanisms. In *NSF Design and Manufacturing Systems Conference*, volume 1, pages 607–611, Charlotte, North Carolina, January 1993.
R. S. Hartenberg and J. Denavit. *Kinematic Synthesis of Linkages*. McGraw-Hill, New York, New York, 1964.
L. Burmester. *Lehrbuch der Kinematik*. LeipZig, 1888.
P. M. Larochelle. On the design of spatial 4c mechanisms for rigid body guidance through four positions. In *Proc. ASME DETC Mechanisms Conf.*, volume DE-82, pages 825–832, Boston, Massachusetts, 1995.
O. Bottema and B. Roth. *Theoretical Kinematics*. North Holland, Amsterdam, 1979.
P.M. Larochelle. *Design of Cooperating Robots and Spatial Mechanisms*. PhD thesis, University of California, Irvine, 1994.
C. H. Suh and C. W. Radcliffe. *Kinematics and Mechanisms Design*. Wiley and Sons, New York, New York, 1978.
L. W. Tsai and B. Roth. Design of dyads with helical, cylindrical, spherical, revolute, and prismatic joints. *Mechanism and Machine Theory*, 7(1):85–102, 1973.
B. Roth. The kinematics of motion through finitely separated positions. *ASME J. of Appl. Mech*, 34:591–598, 1967a.
B. Roth. Finite position theory applied to mechanism synthesis. *ASME J. of Appl. Mech*, 34:599–605, 1967b.
A.P. Murray and J. M. McCarthy. Five position synthesis of spatial *cc* dyads. In *Proc. ASME DETC Mechanisms Conf.*, Minneapolis, Minnesota, September 1994.
B. Roth. On the screw axes and other special lines associated with spatial displacements of a rigid body. *ASME J. of Engr. for Industry*, pages 102–110, February 1967c.
J. Duffy. *Analysis of Mechanisms and Robotic Manipulators*. Wiley and Sons, New York, New York, 1980.
A. Murray and J. McCarthy. A linkage type map for spherical 4 position synthesis. In *Proc. ASME DETC Mechanisms Conf.*, volume DE-82, pages 833–838, 1995.
P. M. Larochelle and J. M. McCarthy. Designing planar mechanisms using a bi-invariant metric in the image space of $so(3)$. *ASME J. of Mechanical Design*, 117(4):646–651, 1996.
Z. Ling and Z. Hu. Use of swept volumes in the design of interference free spatial mechanisms. *Mechanism and Machine Theory*, 32(4):459–476, 1997.