APPROXIMATE MOTION SYNTHESIS USING AN SVD BASED DISTANCE METRIC

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Abstract This paper presents a novel technique for approximate spatial motion generation. Finite spatial locations are represented using homogeneous transforms, which can then be approximated by rotations using products derived from the singular value decomposition. A bi-invariant metric on rotations calculates the *distance* between two locations. A multidimensional nested optimization procedure determines the optimum assembly and associated design variables of a spatial mechanism to minimize the *distances* from the moving body to a finite number of desired locations. The result is a methodology for performing the dimensional synthesis of spatial mechanisms for approximate motion generation. A ten location spatial mechanism design problem is presented to illustrate the method.

Keywords: Kinematic synthesis, spatial mechanisms, motion generation, distance metrics

1. Introduction

In this paper we present a novel dimensional synthesis technique for approximate motion synthesis of spatial linkages. The methodology utilizes an analytic representation of the linkage's workspace that is parameterized by its joint variables. Nonlinear optimization techniques are then employed to minimize the distance from the workspace to a finite number of desired locations. The result is an approximate motion dimensional synthesis technique that is applicable to open or closed spatial kinematic chains. Here, we specifically address the design of spatial 4C mechanisms, see Fig. 1.

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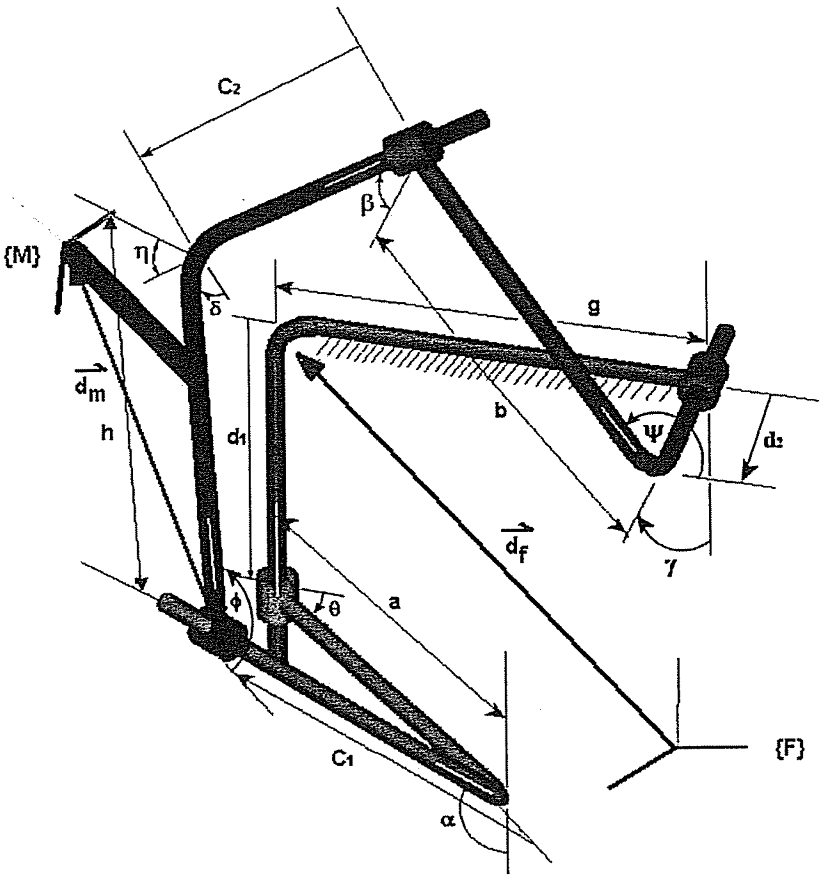

Figure 1. Spatial 4C mechanism

The dimensional synthesis goal is to optimize the design variables such that all of the prescribed locations are either in the workspace or the workspace comes as close as possible to all of the desired locations. Previous works discussing approximate motion synthesis for an arbitrary number of locations include Ravani and Roth, 1983, Bodduluri, 1990, and Larochelle, 1994. In Larochelle, 2000 a methodology for approximate planar motion synthesis that utilizes a parameterized representation of the workspace was reported. That method required a projection of the workspace into the *image space* (see McCarthy, 1990) of planar displacements. Here, we build upon that work and present a general synthesis method that does not require a mapping of the workspace and that is applicable to spatial, spherical, and planar approximate motion

Table 1. Link parameters of the spatial 4C mechanism

Link	*Dual Angle*	*Twist*	*Length*
Driving	$\hat{\alpha}$	α	a
Coupler	$\hat{\eta}$	η	h
Driven	$\hat{\beta}$	β	b
Fixed	$\hat{\gamma}$	γ	g

synthesis. Moreover, we also introduce a novel spatial distance metric which is based upon the use of the singular value decomposition(SVD) of spatial displacements.

2. Spatial 4C Mechanism Workspace

A spatial 4C mechanism has four cylindrical joints, each joint permitting relative rotation and translation along a line, see Fig. 1. The link parameters that define the mechanism are listed in Tbl. 1 and the joint variables are defined Tbl. 2.

A spatial 4C mechanism may be viewed as a combination of two CC dyads. The driving CC dyad has four independent joint variables, referred to as θ, d_1, ϕ and c_1. The driven dyad also has four independent joint variables, ψ, d_2, δ and c_2. When adjoined by the coupler link, the two dyads form a closed chain spatial 4C mechanism with two degrees of freedom. We chose θ and d_1 to be the independent joint variables. Note that ϕ and c_1 as well as the driven dyad's joint variables are now explicit functions of θ and d_1; these functions are found in Larochelle, 1998.

Table 2. Joint variables of the spatial 4C mechanism

Joint	*Dual Angle*	*Rotation*	*Translation*
Driving fixed	$\hat{\theta}$	θ	d_1
Driving moving	$\hat{\phi}$	ϕ	c_1
Driven moving	$\hat{\delta}$	δ	c_2
Driven fixed	$\hat{\psi}$	ψ	d_2

We generate a representation of the workspace that is parameterized by the driving joint variables by performing a forward kinematic analysis of the driving dyad. Fig. 2 illustrates the sequence of transformations in Eq. 1

$$ {}^F_M T = F\ Z(\theta, d_1)\ X(\alpha, a)\ Z(\phi, c_1)\ M \tag{1} $$

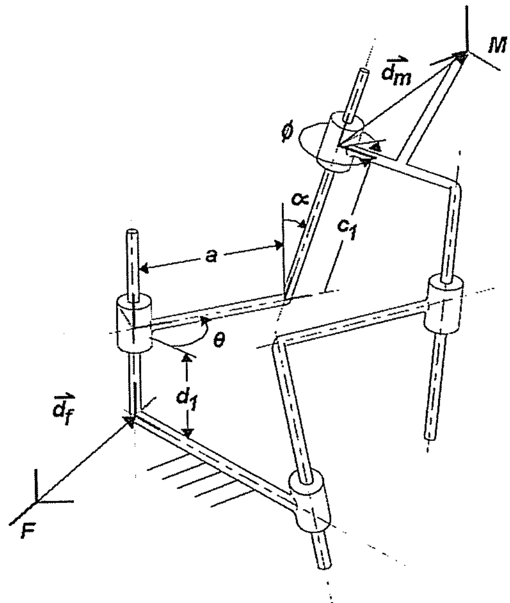

Figure 2. Link coordinate frames

where F and M are the homogeneous transforms from the fixed frame to the first frame of the driving dyad, and from the last frame of the dyad to the moving frame, respectively. Eq. 1 yields the workspace of the spatial 4C mechanisms parameterized by the two driving joint variables θ and d_1 since $\phi(\theta, d_1)$ and $c_1(\theta, d_1)$ are known.

3. SVD Based Distance Metric

We wish to find the optimum design parameters for a rigid body guidance through a finite number of desired locations. The optimization objective function is the sum of the minimum distances between the n desired locations and the workspace of the mechanism; both represented as homogeneous transforms. Hence, we require a metric on spatial displacements. It is well known that no bi-invariant spatial displacement metric exists. However, recently several researchers have made significant contributions towards finding approximately bi-invariant metrics: Chirikjian, 1998, Larochelle, 2000, Gupta, 1997, Martinez and Duffy, 1995, Etzel and McCarthy, 1996, and Kazerounian and Rastegar, 1992. Our approach uses hyper-dimensional rotations to approximate three dimensional displacements. A novel method of approximating spatial motion with rotations in E^4 that uses products derived from the singular value decomposition(SVD) of the homogeneous transform is used, Dees, 2001. We then use biquaternions to represent the rotations in E^4. A bi-invariant metric on biquaternions is then used. The result is an approximately bi-invariant metric on spatial displacements.

A matrix $[P]$ can be factored by using a singular value decomposition:

$$[P] = [U] \begin{bmatrix} s_1 & 0 & \cdots & 0 \\ 0 & s_2 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & s_n \end{bmatrix} [V^T] \quad (2)$$

where the diagonal matrix contains the singular values of $[P]$ and $[U]$ & $[V]$ are either rotations or reflections. The following algorithm (based upon related work by Hanson and Norris, 1981) uses the SVD to construct a 4×4 rotation matrix given a 4×4 homogeneous transform.

Step 1: Find the SVD of the homogeneous transform [T]:

$$[T] = [U]\ [diag(s_1, s_2, ..., s_n)]\ [V]^T,$$

where $n = 4$.

Step 2: Find the determinant of $[X]$:

$$\sigma = det(x).$$

where

$$[X] = [U]^T\ [V].$$

Step 3: Construct $[A]$, the 4×4 rotation matrix:

$$[A] = [U]\ [diag(1, 1, 1, \sigma)]\ [V]^T,$$

where $[A]$ is a rotation in E^4.

3.1 Distance between 4 × 4 Rotations

Ge's (1994) construction is used to obtain the biquaternions representing the two 4×4 rotation matrices, see Ge, 1994 and Dees, 2001. A bi-invariant metric on biquaternions is:

$$d(\hat{Q}, \hat{R}) = \sqrt{(Q - R)^T(Q - R) + (S - T)^T(S - T)} \quad (3)$$

where $\hat{Q} = (Q, S)$ and $\hat{R} = (R, T)$ are biquaternions, and Q, S, R, and T are the associated quaternions. Etzel and McCarthy, 1996 prove that this metric is bi-invariant.

4. Approximate Motion Synthesis

The goal here is dimensional synthesis of spatial 4C mechanisms for approximate motion synthesis. The $(20+2n)$ dimensional design vector is

$$\vec{r} = \left[\alpha\ a\ \gamma\ g\ \beta\ b\ \eta\ h\ \vec{d}_m\ lng_m\ lat_m\ rol_m\ \vec{d}_f\ lng_f\ lat_f\ rol_f\ \vec{\theta}^T\ \vec{d_1}^T\right]^T \tag{4}$$

where the first eight elements are the link parameters(see Fig. 1 and Tbl. 1). Next are the terms which define F and M; the location of the mechanism with respect to the fixed frame and the location of the moving frame with respect to the coupler link frame. The 3×1 column vectors $\vec{d}_*$ and the angles lng_*, lat_* and rol_* define the position and orientations of these two transformations, respectively. Last are the n dimensional vectors $\vec{\theta}$ and $\vec{d_1}$. These joint pairs are optimized for each of the n desired locations.

The nested optimization strategy is as follows:

1 A candidate design (i.e. the first 20 components of $\vec{r}$) is determined.

2 Utilizing the techniques of Murray and Larochelle, 1998 verify that the candidate solution has a crank-rocker spherical image-Larochelle, 2000 showed that such mechanisms will not suffer from rotational or translational singular configuration defects.

3 The steps below are performed for each of the two circuits of the candidate solution:

   (a) For each desired location the values of θ and d_1 are optimized by utilizing the svd based distance metric. Note that an optimized θ and d_1 pair determine the location in the mechanism's workspace nearest a desired location. This is the inner optimization.

   (b) The sum of the distances to each of the n desired locations is determined.

4 The distance sums of the two circuits of the mechanism are compared; and the lesser is utilized. If the distance sum is acceptable then the design is complete. Otherwise, the distance sum and the first 20 components of $\vec{r}$ are sent to the outer optimization to determine a better candidate design and the above steps are repeated.

5. Design Case Study

We address a 10 location rigid body guidance problem taken from Larochelle, 1994. The desired locations (#) and the locations in the workspace of the solution 4C mechanism nearest them (#') are listed in Tbl. 3. The joint variables that guide the moving body to these locations as well as the distance between these locations to each of the desired locations are also listed. The sum of these distances (3.53E-1) is the minimum value of the objective function found and corresponds to the total distance from the workspace of this solution mechanism to the ten desired locations. The dimensional synthesis parameters of the

Table 3. Spatial 4C synthesis results for 10 locations

Loc.	*X*	*Y*	*Z*	*Lng.*	*Lat.*	*Rol*	θ	d_1	*Dist.*
1	1.0	0.0	5.0	100.0	0.0	0.0			
1'	-0.85	0.22	1.71	54.6	11.8	-0.20	-24.8	7.0	8.26E-2
2	2.0	0.0	4.0	90.0	0.0	10.0			
2'	-0.56	0.42	2.61	55.8	17.2	-0.5	-12.1	6.6	4.18E-2
3	3.0	0.0	3.0	80.0	0.0	20.0			
3'	0.12	0.76	3.41	58.0	25.1	0.3	13.2	5.7	4.49E-2
4	4.0	0.0	2.0	70.0	0.0	30.0			
4'	1.04	0.81	3.28	60.5	33.2	2.8	7.2	3.5	3.50E-2
5	5.0	0.0	1.0	60.0	0.0	40.0			
5'	1.40	0.05	2.12	61.3	48.3	18.7	-11.1	-2.6	6.63E-2
6	6.0	0.0	-1.0	50.0	0.0	50.0			
6'	4.97	-0.40	-0.56	45.1	-7.4	42.7	-1.5	1.5	2.48E-2
7	7.0	0.0	-2.0	40.0	0.0	60.0			
7'	5.18	-0.09	-1.45	37.4	-3.6	56.8	-14.5	5.4	1.85E-2
8	8.0	0.0	-3.0	30.0	0.0	70.0			
8'	6.43	0.03	-2.44	25.0	-0.7	69.9	22.8	9.6	1.68E-2
9	9.0	0.0	-4.0	20.0	0.0	80.0			
9'	8.94	-0.02	-3.94	19.9	0.4	80.3	10.0	15.5	2.30E-2
10	10.0	0.0	-5.0	10.0	0.0	90.0			
10'	12.89	0.11	-6.75	14.0	0.5	87.1	9.8	25.0	2.01E-2

solution 4C mechanism, i.e. the first 20 components of $\vec{r}$, are: 149.9, -2.31, 88.2, 0.81, 72.6, -0.25, 49.2, -2.89, -2.40, -0.17, 1.28, 118.2, 226.6, -15.8, 1.07, 0.70, 1.28, 26.4, 133.8, 276.3.

6. Conclusions

We have presented our development of a new method for approximate motion synthesis for rigid body guidance through n desired spatial locations. This method utilizes a parameterized representation of the mechanism's workspace that is exploited to yield mechanisms that will not suffer from rotational or translational singular configuration defects. Moreover, this dimensional synthesis procedure employs a novel approximately bi-invariant svd-based distance metric. The result is a nested multidimensional direct search optimization strategy that yields the optimal design parameters such that the mechanism comes as close as possible to all of the n desired locations in a single circuit or assembly.

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