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APPROXIMATE MOTION SYNTHESIS OF SPHERICAL KINEMATIC CHAINS

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ABSTRACT

In this paper we present a novel dimensional synthesis technique for approximate motion synthesis of spherical kinematic chains. The methodology uses an analytic representation of the spherical RR dyad's workspace that is parameterized by its dimensional synthesis variables. A two loop nonlinear optimization technique is then employed to minimize the distance from the dyad's workspace to a finite number of desired orientations of the workpiece. The result is an approximate motion dimensional synthesis technique that is applicable to spherical open and closed kinematic chains. Here, we specifically address the spherical RR open and 4R closed chains however the methodology is applicable to all spherical kinematic chains. Finally, we present two examples that demonstrate the utility of the synthesis technique.

INTRODUCTION

The novel dimensional synthesis technique presented utilizes an analytic representation of the spherical RR dyad's workspace that is parameterized by its dimensional synthesis variables. The parameterized workspace represents the geometric constraint imposed on the motion of the moving body or workpiece. This constraint is a result of the geometric and kinematic structure of the dyad; e.g. its length and the location of its fixed and moving axes (i.e. lines). The workspace is an analytical representation of the workspace of the dyad that is parameterized by the dyad's dimensional synthesis variables. Here we derive the parameterized workspace of spherical RR dyads using 3×3 elements of SO(3) (also known as rotation matrices) and utilize this representation to perform dyadic dimensional synthesis for approximate rigid body guidance.

The derivation of the parameterized workspace involves writing the kinematic constraint equations of the dyad using homogeneous coordinate transformations. The result is an analytical representation of the workspace of the dyad that is parameterized by its joint variables. The synthesis goal is to vary the design variables such that all of the prescribed locations are either: (1) in the workspace, or, (2) the workspace comes as close as possible to all of the desired locations. Recall that in general five is the largest number of locations for which an exact solution is possible for the spherical RR dyad, see [1, 2].

In related works, kinematic mappings have been used to derive the constraint manifold representation of the kinematic constraint equations. The derivation of the constraint manifold involves writing the kinematic constraint equations using the image space representation of displacements, see [3], [4], and [5]. In [6] the constraint manifold of the spherical RR dyad is used to solve the 5 orientation Burmester problem. Previous works discussing constraint manifold fitting for an arbitrary number of locations include [7], [8], [9], and [4]. All of these works employ *implicit* representations of the dyad constraint manifolds. The constraint manifolds, that are known to be highly nonlinear [10], are approximated by tangent hyperplanes by using a standard Taylor series linearization strategy. The distance from

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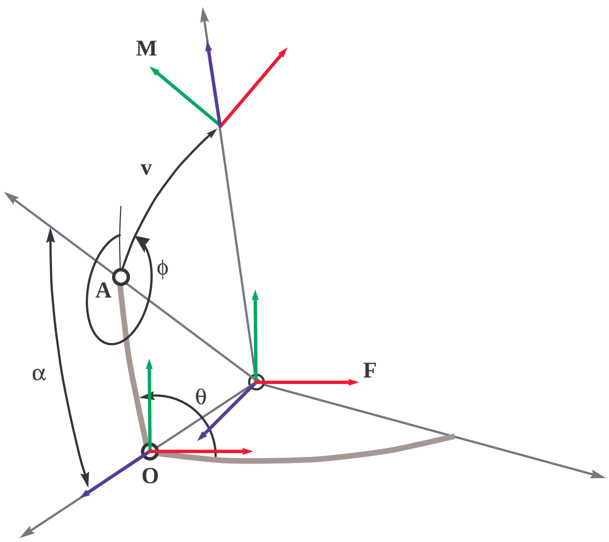

Figure 1. A SPHERICAL RR OPEN CHAIN.

the approximating tangent plane to the desired location is then used to formulate an objective function to be minimized. These efforts met with limited success since the constraint manifolds are highly nonlinear and the approximating tangent planes yield poor measures of the distance from the constraint manifold to the desired locations, [11] and [4]. In [12, 13] work that addressed the synthesis of planar RR dyads via parameterized constraint manifold fitting was reported. Here we build upon that work and utilize parameterized workspaces and employ nonlinear optimization to yield a general numerical dimensional synthesis technique for approximate motion synthesis.

We proceed by deriving the parameterized workspaces of spherical RR open and 4R closed kinematic chains. This is followed by the kinematic analyses utilized to yield closed-chain design solutions that do not suffer from circuit defects. Finally, we present the general approximate motion synthesis procedure and two numerical examples.

SPHERICAL RR WORKSPACE

In this section we derive the parametric form of the workspace of the spherical RR dyad. The workspace is derived by expressing analytically the geometric structure that the joints of the dyad impose on the moving body. Using 3×3 rotation matrices and the geometric constraint equations of the dyad we arrive at a representation of the workspace that is parameterized by the dimensional synthesis variables of the dyad.

The RR dyad, shown in Fig. 1, is a two degree of freedom open chain with one fixed R joint, one moving R joint, and whose link length is given by the angle α between the two joint axes. The fixed R joint is located by $\mathbf{u}$ measured with respect to the fixed frame $\mathbf{F}$ and the moving frame $\mathbf{M}$ is given by $\mathbf{v}$ measured with respect to a frame attached to the floating link whose origin is at $\mathbf{A}$. The displacement from the fixed frame $\mathbf{F}$ to $\mathbf{O}$ is given by the vector $\mathbf{u}$, where $\mathbf{u} = [u_{long}\ u_{lat}]^T$ are the longitude and latitude angles that orient $\mathbf{u}$ with respect to $\mathbf{F}$. The displacement from $\mathbf{A}$ to $\mathbf{M}$ is similarly given by the vector $\mathbf{v}$, where $\mathbf{v} = [v_{long}\ v_{lat}]^T$. The forward kinematics of the spherical RR dyad from the fixed frame $\mathbf{F}$ to the moving frame $\mathbf{M}$ may be written as:

$$
{}^{F}_{M}[R] = \left[rot_y(u_{long})\right][rot_x(-u_{lat})][rot_z(\theta)]\ldots \\
[rot_y(\alpha)][rot_z(\phi)]\left[rot_y(v_{long})\right][rot_x(-v_{lat})] \quad (1)
$$

where, rot_y, rot_x, rot_z are elements of SO(3) representing rotations about the Y, X, and Z axes respectively. Note that the joint variables of the dyad are θ and ϕ. The remaining parameters in Eqn. (1) are the dyad's dimensional synthesis variables. We now group these variables and define the design vector $\mathbf{r}$ for the spherical RR dyad as: $\mathbf{r} = [u_{long}\ u_{lat}\ \alpha\ v_{long}\ v_{lat}]^T$. Finally, for a given dyad defined by $\mathbf{r}$, the parameterized workspace Υ from the fixed frame $\mathbf{F}$ to the moving frame $\mathbf{M}$ may be written as:

$$
\Upsilon(\theta,\phi) = \left[rot_y(u_{long})\right][rot_x(-u_{lat})][rot_z(\theta)]\ldots \\
[rot_y(\alpha)][rot_z(\phi)]\left[rot_y(v_{long})\right][rot_x(-v_{lat})]. \quad (2)
$$

SPHERICAL 4R WORKSPACE

In this section we derive the parametric form of the workspace of the spherical 4R closed chain, also known as the spherical four-bar mechanism. A spherical 4R mechanism consisting of two RR dyads, one of length α the other β, is shown in Fig. 2. We proceed in a manner similar to that taken for the spherical RR dyad and derive the parameterized workspace of the spherical 4R mechanism.

The 4R mechanism is a one degree of freedom closed chain with two fixed R joints and two moving R joints. The fixed R joints are located by $\mathbf{u_1}$ and $\mathbf{u_2}$ measured with respect to the fixed frame $\mathbf{F}$ and the moving frame $\mathbf{M}$ is given by $\mathbf{v}$ measured with respect to a frame attached to the floating link whose origin is on the moving axis of dyad #1 and whose z axis is aligned with the joint axis. We define a fixed frame $\mathbf{G}$ such that its z-axis is along $\mathbf{u_1}$ and its y-axis is orthogonal to the $\mathbf{u_1}$ and $\mathbf{u_2}$ plane. The transformation from the fixed frame $\mathbf{F}$ to $\mathbf{G}$ is then given by,

$$
[G] = [\hat{x}\ \hat{y}\ \hat{z}] \quad (3)
$$

where,

$$
\hat{z} = \mathbf{u_1}
$$

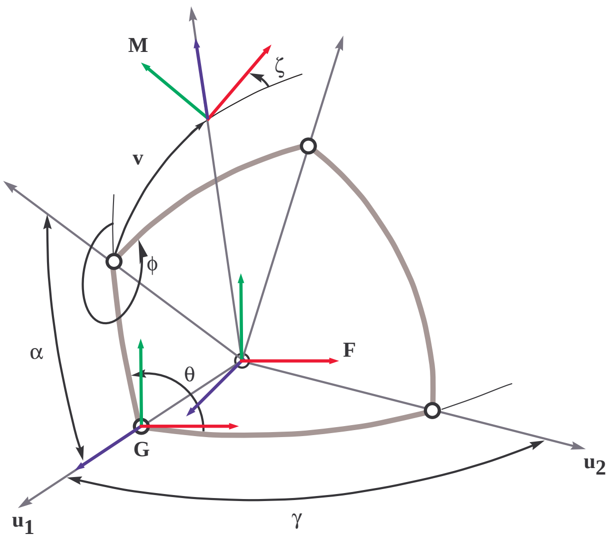

Figure 2. A SPHERICAL 4R CLOSED CHAIN.

$$\hat{y} = \mathbf{u_1} \times \mathbf{u_2}$$
$$\hat{x} = \hat{y} \times \hat{z}$$

The forward kinematics of the spherical 4R mechanism from the fixed frame **F** to the moving frame **M** may be written as:

$$ {}^F_M[R] = [G]\,[rot_z(\theta)]\,[rot_y(\alpha)]\,[rot_z(\phi)]\ldots \\ \left[rot_y(v_{long})\right][rot_x(-v_{lat})]\,[rot_z(\zeta)] \quad (4)$$

Note that the single joint variable of the mechanism is θ since ϕ is a known function of θ from the kinematic analysis of the closed chain, see [1]. The remaining parameters in Eqn. (4) are the dyad's dimensional synthesis variables. We now group these variables and define the design vector **r** for the spherical 4R mechanism as: $\mathbf{r} = [\mathbf{u_1}^T\ \mathbf{u_2}^T\ \alpha\ \eta\ \beta\ v_{long}\ v_{lat}\ \zeta]^T$. Finally, for a given mechanism defined by **r**, the parameterized workspace Υ from the fixed frame **F** to the moving frame **M** may be written as:

$$\Upsilon(\theta) = [G]\,[rot_z(\theta)]\,[rot_y(\alpha)]\,[rot_z(\phi)]\ldots \\ \left[rot_y(v_{long})\right][rot_x(-v_{lat})]\,[rot_z(\zeta)]\,. \quad (5)$$

METRIC ON SO(3)

In order to perform the approximate motion synthesis we require a metric on SO(3) to define the distance between elements of the workspace of the chain and the desired orientations.

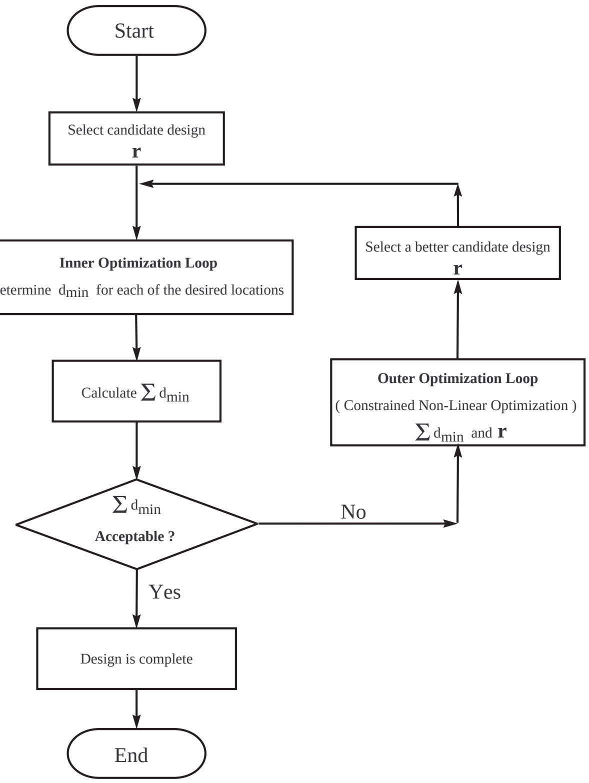

Figure 3. FLOW CHART.

Many useful bi-invariant metrics on SO(3) have been proposed, see [4, 7, 14–19]. Here, we chose to use the metric proposed in [20] where the distance d between two elements $[A_1]$ and $[A_2]$ of SO(3) is defined by,

$$d = \|[I] - [A_2][A_1]^T\|_F \quad (6)$$

where F denotes the Frobenius norm.

APPROXIMATE MOTION SYNTHESIS

The design problem can be stated as follows: given a finite set of n desired orientations the task is to determine the open or closed chain that guides the work piece through, or as close as possible, to these orientations. Our approach is to utilize the metric discussed above to determine the distance from the workspace to each of the n desired locations, sum these distances, and then to employ nonlinear optimization techniques to vary the dimensional synthesis parameters such that the total distance is minimized. We utilize a two loop optimization approach, see Fig. 3.

Table 1. FOUR DESIRED ORIENTATIONS.

#	*Long*	*Lat*	*Roll*
1	−35.0	35.0	90.0
2	30.0	37.0	30.0
3	15.0	50.0	45.0
4	−15.0	45.0	90.0

Inner Optimization Routine

For a given design **r** the inner loop optimization seeks to determine the joint angles such that the distance from the workspace to the n desired rigid body orientations is minimized. An exhaustive direct search of the parameterized workspace is performed utilizing Eqn. (6). Note that in the case of the RR dyad the joint space is two dimensional and in the case of the 4R closed chain it is one dimensional. Hence, exhaustive direct searches over the parameterized workspace are also only one or two dimensional. The result of the exhaustive direct search is the minimum distance d_{min} from the workspace to each of the n desired orientations.

Outer Optimization Routine

The outer optimization routine seeks to find the dimensional synthesis variables (i.e. **r**) that minimizes the sum of the distances to each of the desired orientations. We utilize the constrained nonlinear minimization routine `fmincon()` from the MATLAB Optimization Toolbox. Constraints are used to impose conditions on the geometry of the chain, for instance on the link lengths, fixed axis locations, joint limits, etc. Constraints may also used to restrict closed chain designs to a desired type of mechanism such as a Grashof crank-rocker. Moreover, constraints may be added to avoid circuit defects. This is easily accomplished since a parameterized representation of the workspace is being utilized. At each iteration of the outer optimization loop the candidate design **r** is analyzed to determine if it is Grashof or not. For Grashof mechanisms the inner optimization is performed twice, once for each circuit. For each circuit the sum of the d_{min} is determined. The circuit with the minimum sum is utilized to determine if the design is acceptable or not.

EXAMPLE: SPHERICAL RR DYAD

Here, we perform the dimensional synthesis of a spherical RR dyad to reach four desired orientations defined in Tab. 1 and shown on the design sphere in Fig. 4 where orientations $[A]$ have been defined using the longitude, latitude, and roll convention: $[A] = [rot_y(long)]\,[rot_x(-lat)]\,[rot_z(roll)]$. The initial guess and the optimized parameters obtained are shown in Table 2. The joint angles for the synthesized dyad and the distances associated with the various orientations are shown in Tab. 3. Fig. 5 shows the optimal design. The path shown is a linear interpolation of the joint angles θ and ϕ through the desired orientations.

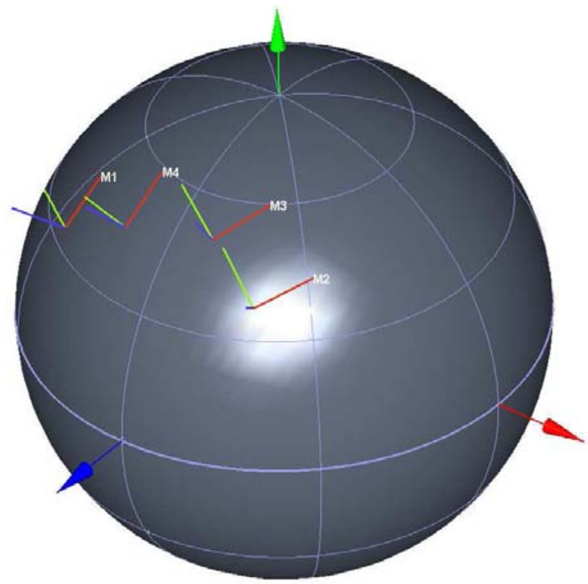

Figure 4. FOUR DESIRED ORIENTATIONS.

Table 2. THE OPTIMAL RR DESIGN.

Parameter	*Initial Guess*	*Solution*
u_{long}	25	1.8761
u_{lat}	25	5.2349
α	70	19.9748
v_{long}	20	20.3284
v_{lat}	20	18.9405

Table 3. JOINT PARAMETERS AND DISTANCES.

#	θ	ϕ	d_{min}
1	156.0	280.0	0.0001
2	16.9	16.0	0.0002
3	59.0	346.5	0.0111
4	82.7	−6.5	0.065
			$\Sigma = 0.0763$

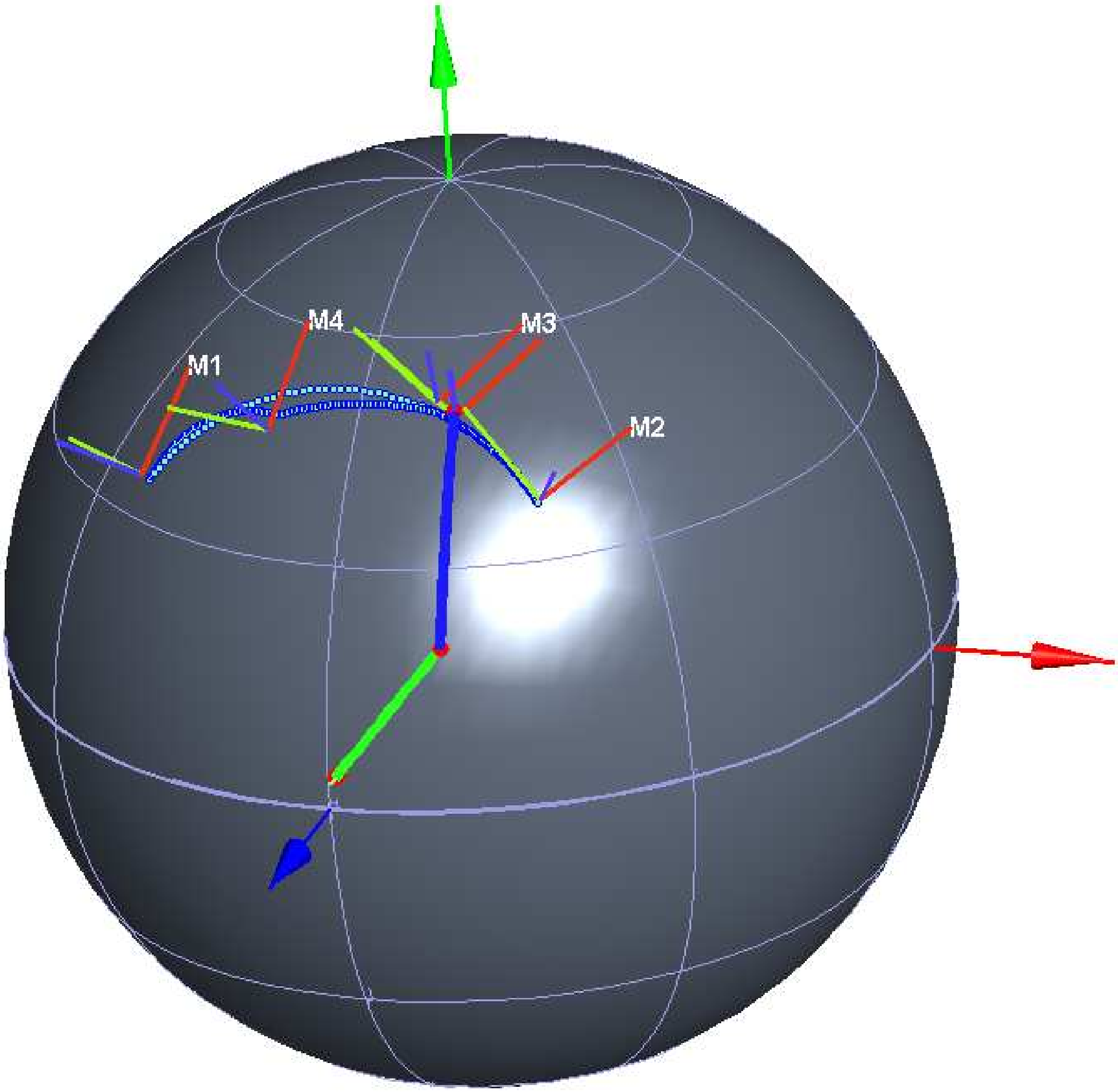

Figure 5. THE OPTIMAL RR DESIGN.

EXAMPLE: SPHERICAL 4R MECHANISM

Here we design a 4R mechanism to reach six desired orientations defined in Tab. 4 and shown on the design sphere in Fig. 6 The optimized parameters obtained for this case are shown in Tab. 5. The joint angles for the synthesized 4R mechanism and the distances associated with the orientations are shown in Tab. 6.

Fig. 7 shows the optimal design. The path shown is the coupler curve for the 4R mechanism. The spherical 4R mechanism is a crank-rocker that attains all the desired orientations in a single circuit and thus does not suffer from a circuit defect. Fig. 8 shows the design nearest the fifth orientation.

CONCLUSIONS

In this paper we have presented a novel dimensional synthesis technique for approximate motion synthesis of spherical kinematic chains. The methodology uses an analytic representation of the spherical RR dyad's workspace that is parameterized by its dimensional synthesis variables. A two loop nonlinear optimization technique is then employed to minimize the distance from the dyad's workspace to a finite number of desired orientations of the workpiece. The result is an approximate motion dimensional synthesis technique that is applicable to spherical open and closed kinematic chains. Though the cases of spherical

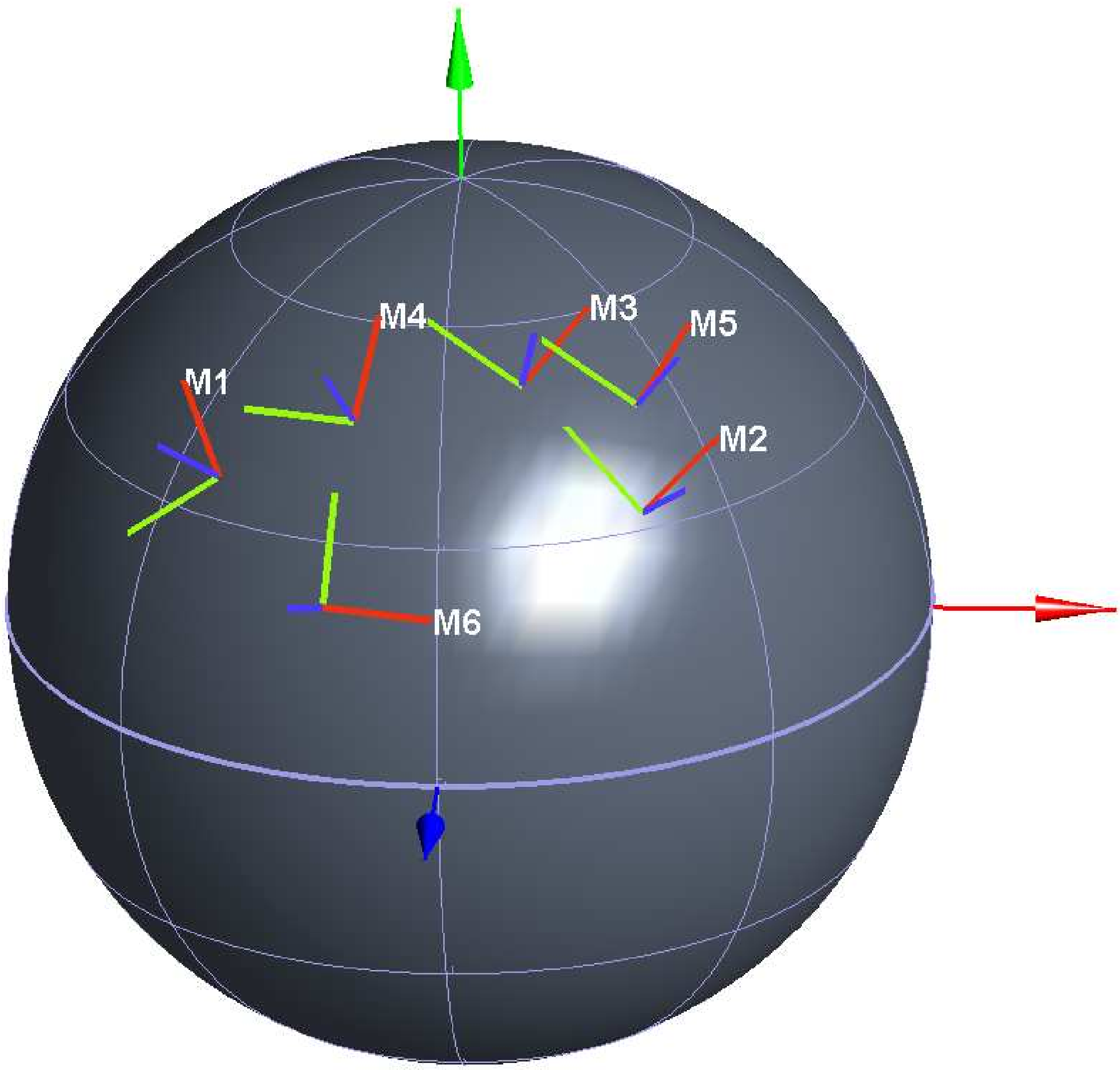

Figure 6. SIX DESIRED ORIENTATIONS.

RR open and 4R closed chains were addressed specifically, the methodology is applicable to all spherical kinematic chains. Two examples that demonstrated the utility of the synthesis technique were presented.

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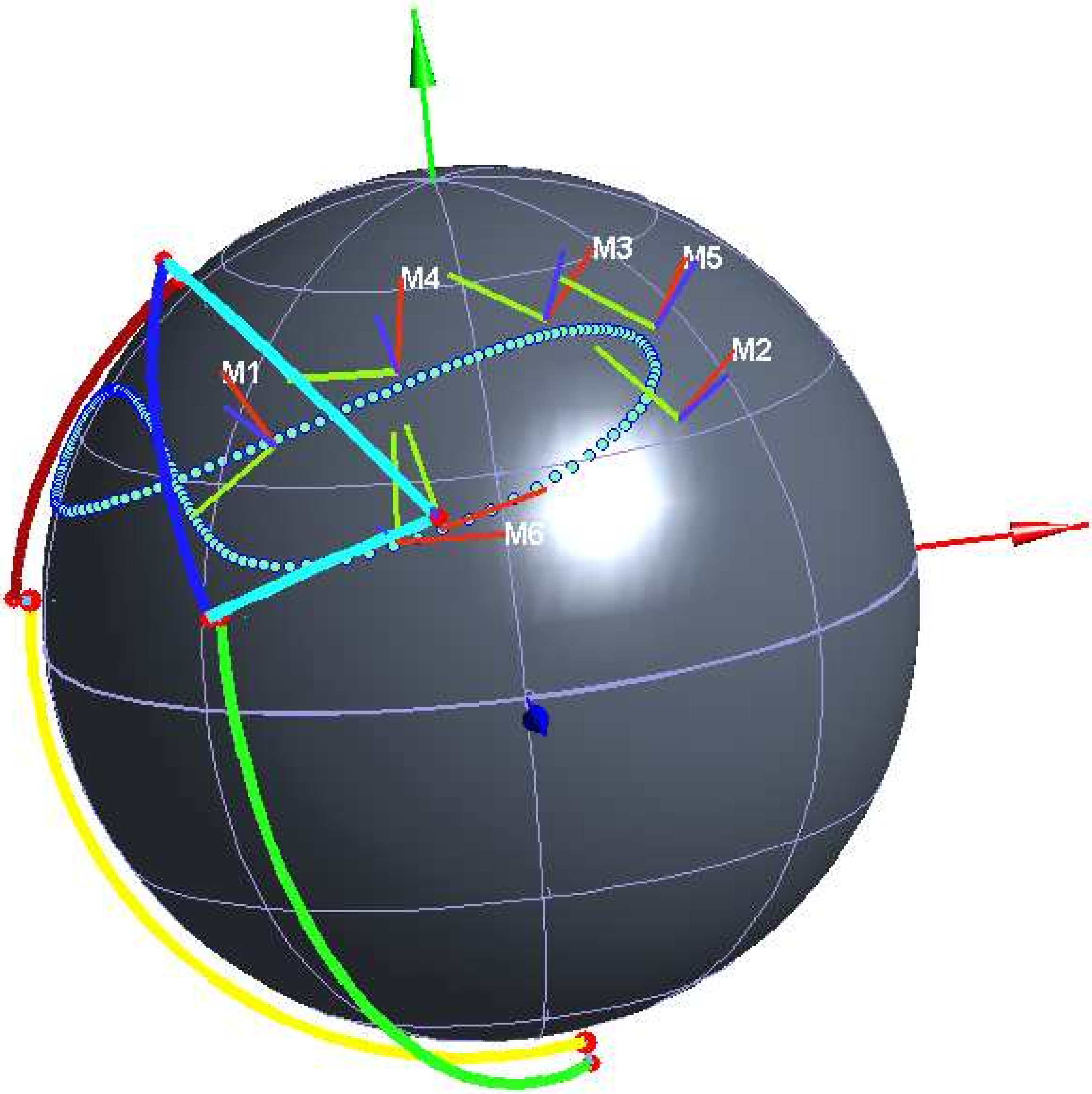

Figure 7. THE OPTIMAL 4R DESIGN.

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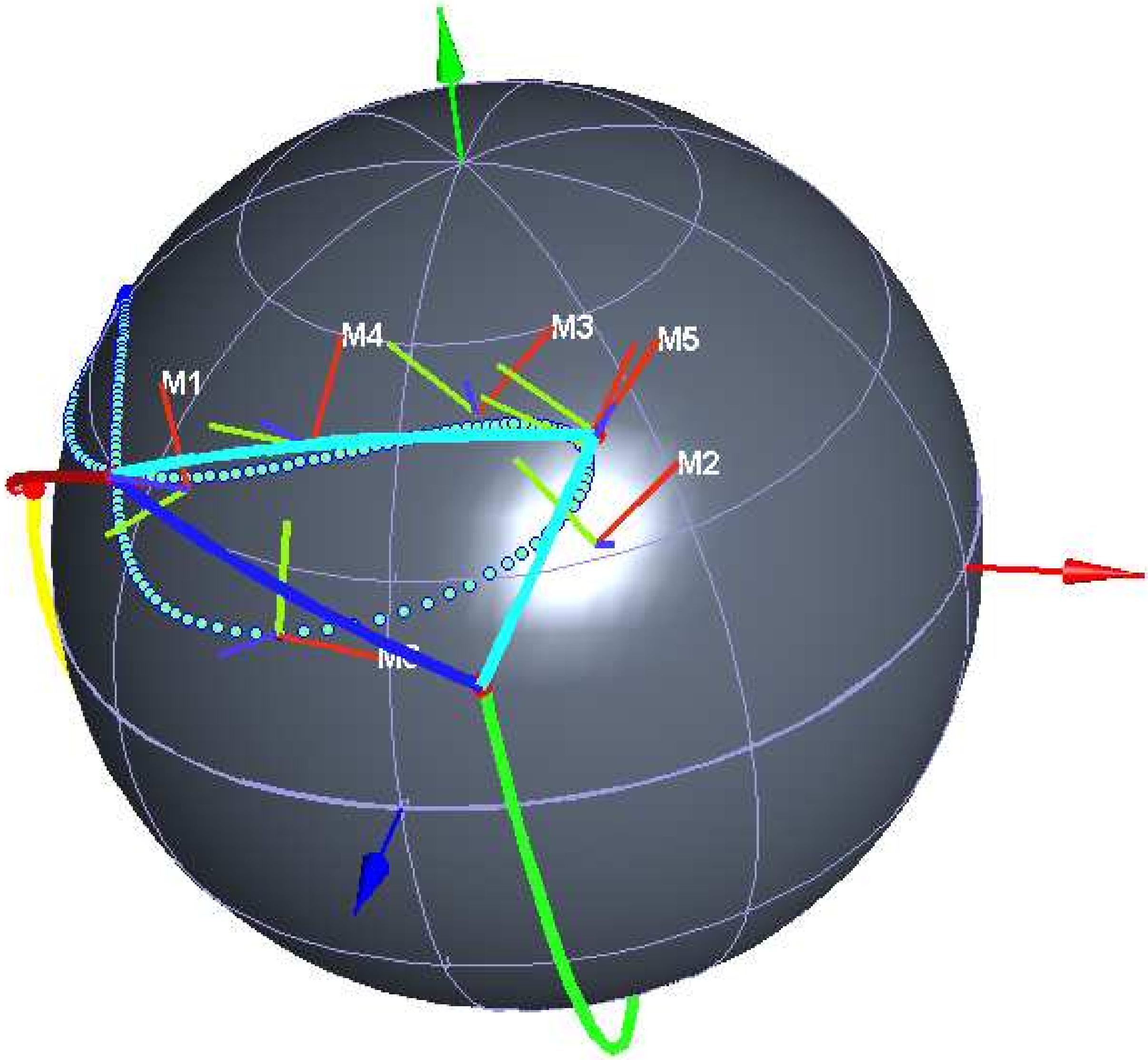

Figure 8. NEAREST THE FIFTH ORIENTATION.

Table 4. SIX DESIRED ORIENTATIONS.

#	*Long*	*Lat*	*Roll*
1	−35.0	35.0	135.0
2	30.0	32.0	30.0
3	15.0	50.0	45.0
4	−15.0	45.0	90.0
5	35.0	45.0	37.5
6	−15.0	22.0	0.0

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Table 5. THE OPTIMAL 4R DESIGN.

Parameter	*Initial Guess*	*Solution*
u_{1-long}	-18.2337	-83.8800
u_{1-lat}	-18.7504	9.4027
u_{2-long}	15.6723	76.7050
u_{2-lat}	-26.2359	-82.8900
α	15.4402	45.5370
η	27.7729	50.4944
β	40.5100	105.6410
v_{long}	66.9697	51.5159
v_{lat}	40.4873	29.8364
ζ	6.8365	87.5200

Table 6. JOINT PARAMETERS AND DISTANCES.

#	θ	ϕ	d_{min}
1	49.10	76.67	0.1678
2	144.40	-56.20	0.1211
3	139.50	-46.85	0.3901
4	95.10	16.48	0.7082
5	133.50	-36.39	0.1225
6	176.80	-135.40	0.3030
			$\sum = 1.8127$

anism design". *ASME Journal of Mechanical Design,* **122**(4), December, pp. 457–463.
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