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PROJECTION METRICS FOR RIGID-BODY DISPLACEMENTS

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ABSTRACT

An open research question is how to define a useful metric on SE(n) with respect to (1) the choice of coordinate frames and (2) the units used to measure linear and angular distances. We present two techniques for approximating elements of the special Euclidean group SE(n) with elements of the special orthogonal group SO(n+1). These techniques are based on the singular value and polar decompositions (denoted as SVD and PD respectively) of the homogeneous transform representation of the elements of SE(n). The projection of the elements of SE(n) onto SO(n+1) yields hyperdimensional rotations that approximate the rigid-body displacements (hence the term *projection metric*. A bi-invariant metric on SO(n+1) may then be used to measure the *distance* between any two spatial displacements. The results are PD and SVD based projection metrics on SE(n). These metrics have applications in motion synthesis, robot calibration, motion interpolation, and hybrid robot control.

INTRODUCTION

Simply stated a metric measures the distance between two points in a set. There exist numerous useful metrics for defining the distance between two points in Euclidean space, however, defining similar metrics for determining the distance between two locations of a finite rigid body is still an area of ongoing research, see [22], [14], [2], [26], [23], [21], [8], [11], [30], [4], [7], and [1]. In the cases of two locations of a finite rigid body in either SE(3) (spatial locations) or SE(2) (planar locations) any metric used to measure the distance between the locations yields a result which depends upon the chosen reference frames, see [2] and [23]. However, a metric that is independent of these choices, referred to as being bi-invariant, is desirable. It is well known that for the specific case of orienting a finite rigid body in SO(n) bi-invariant metrics do exist. For example, Ravani and Roth [27] defined the distance between two orientations of a rigid body in space as the magnitude of the difference between the associated quaternions and a proof that this metric is bi-invariant may be found in [21].

In [21] and [18] Larochelle and McCarthy presented an algorithm for approximating displacements in SE(2) with orientations in SO(3). By utilizing the metric of Ravani and Roth [27] they arrived at an approximate bi-invariant metric for planar locations in which the error induced by the spherical approximation is of the order $\frac{1}{\mathsf{R}^2}$, where R is the radius of the approximating sphere. Their algorithm for a projection metric is based upon an algebraic formulation which utilizes Taylor series expansions of $sine()$ and $cosine()$ terms in homogeneous transforms, see [24]. Etzel and McCarthy [8] extended this work to spatial displacements by using orientations in $SO(4)$ to approximate locations in $SE(3)$. Their algorithm is also based upon Taylor series expansions of $sine()$ and $cosine()$ terms, see [10], and here too the error is of the order $\frac{1}{\mathsf{R}^2}$.

This paper presents an efficient alternative approach for defining projection metrics on SE(n) to those presented by Larochelle and McCarthy [21] and Etzel and McCarthy [8].

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Here, the underlying geometrical motivations are the same- to approximate displacements with hyperspherical rotations. However, an alternative approach for reaching the same goal is presented. We utilize the singular value and polar decompositions to yield projections of planar and spatial finite displacements onto hyperspherical orientations.

PROJECTING SE(n) ONTO SO(n+1)

First, we review how spherical displacements may be used to approximate planar displacements with some finite error associated with the radius R of the sphere, see [15] and [21]. This approach is based upon the work of McCarthy [24] in which he examined spherical and 3-spherical motions with instantaneous invariants approaching zero and showed that these motions may be identified with planar and spatial motions, respectively.

Recall that a general planar displacement (a, b, α) in the $z = \mathsf{R}$ plane (an element of SE(2)) may be expressed as a homogeneous coordinate transformation,

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = [A_p] \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{bmatrix} \cos\alpha & -\sin\alpha & a \\ \sin\alpha & \cos\alpha & b \\ 0 & 0 & \mathsf{R} \end{bmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}. \quad (1)$$

Now consider a general spherical displacement in which the parameters used to describe the displacement are the three angles longitude(θ), latitude(ϕ), and roll(ψ), see Fig. 1. Using these parameters a general spherical displacement may be written as,

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = [A_s] \begin{pmatrix} x \\ y \\ z \end{pmatrix} = Rot(y, \theta) Rot(x, -\phi) Rot(z, \psi) \begin{pmatrix} x \\ y \\ z \end{pmatrix}. \quad (2)$$

We now define $\hat{a} = \mathsf{R}\theta$ as the longitudinal arc length and $\hat{b} = \mathsf{R}\phi$ as the latitudinal arc length. If we consider displacements in the $z = \mathsf{R}$ plane and expand the trigonometric functions *sine*() and *cosine*() using a Taylor series about 0 and substitute the angles θ and ϕ from above into the expansions then we may rewrite Eq. 2 as,

$$\begin{aligned} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = & \begin{bmatrix} \cos\psi & -\sin\psi & \hat{a} \\ \sin\psi & \cos\psi & \hat{b} \\ 0 & 0 & \mathsf{R} \end{bmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} \\ & + \frac{1}{\mathsf{R}} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -\hat{a}\cos\psi - \hat{b}\sin\psi & \hat{a}\sin\psi - \hat{b}\cos\psi & -\frac{1}{2}(\hat{a}^2 + \hat{b}^2) \end{bmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} \\ & + O(\frac{1}{\mathsf{R}^2}). \end{aligned} \quad (3)$$

Note that the first term of Eq. 3 is identical to Eq. 1 and we may approximate planar displacements (a, b, ψ) with some finite error that is associated with the radius of the sphere. From Eq. 3 we make the following identifications: $\hat{a} \Rightarrow a$, $\hat{b} \Rightarrow b$, and, $\psi \Rightarrow \alpha$. Using the definition of the arc lengths and the radius of the sphere we obtain the three angles; θ, ϕ, and ψ, which describe the spherical displacement on the sphere of radius R that approximates the prescribed planar displacement: $\theta = \frac{a}{\mathsf{R}}$, $\phi = \frac{b}{\mathsf{R}}$, and, $\psi = \alpha$.

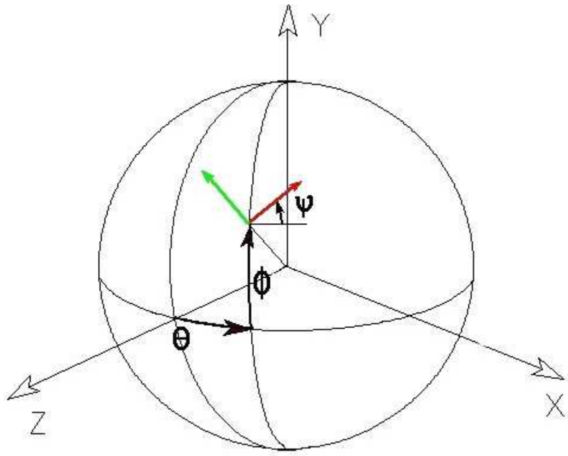

Figure 1. Planar Case: SE(2) ⇒ SO(3)

Etzel and McCarthy [8] extended the above methodology to spatial displacements by using orientations in SO(4) to approximate locations in SE(3). They showed that a 4x4 homogeneous transform representation of SE(3) can be approximated by a pure rotation $[D]$ in SO(4),

$$[D] = [J(\alpha, \beta, \gamma)][K(\theta, \phi, \psi)] \quad (4)$$

where,

$$J(\alpha, \beta, \gamma)] = \begin{bmatrix} \cos\alpha & 0 & 0 & \sin\alpha \\ -\sin\beta\sin\alpha & \cos\beta & 0 & \sin\beta\cos\alpha \\ -\sin\gamma\cos\beta\sin\alpha & -\sin\gamma\sin\beta & \cos\gamma & \sin\gamma\cos\beta\cos\alpha \\ -\cos\gamma\cos\beta\sin\alpha & -\sin\beta\cos\gamma & -\sin\gamma & \cos\gamma\cos\beta\cos\alpha \end{bmatrix}$$

and,

$$K(\theta, \phi, \psi)] = \begin{bmatrix} & & & 0 \\ & [A_s] & & 0 \\ & & & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

The angles α, β and γ are defined as follows: $\tan(\alpha) = \frac{d_x}{\mathsf{R}}$, $\tan(\beta) = \frac{d_y}{\mathsf{R}}$, and $\tan(\gamma) = \frac{d_z}{\mathsf{R}}$ where d_x, d_y, and d_z are the components of the translation vector $\mathbf{d}$ of the displacement and R is the radius of the hypersphere. A conceptual representation, analogous to Fig. 1, can be seen in Fig. 2.

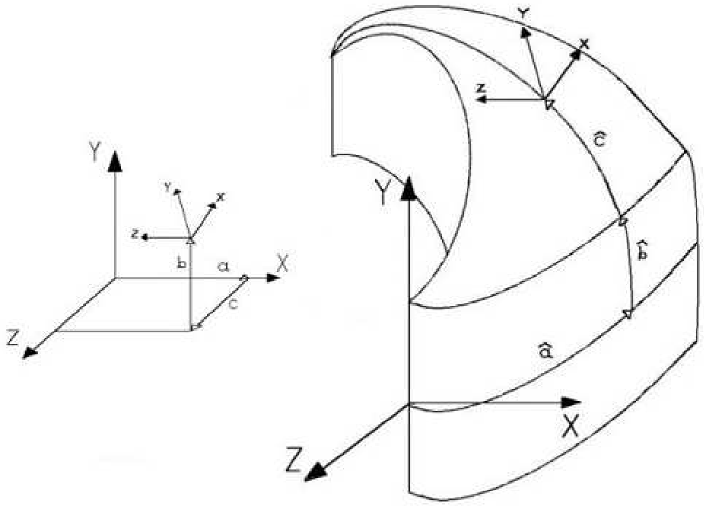

Figure 2. Spatial Case: SE(3) ⇒ SO(4) (figure from [24])

THE SVD BASED PROJECTION

The SVD-based projection metric also uses hyperdimensional rotations to approximate displacements. However, this technique uses products derived from the singular value decomposition (SVD) of the homogeneous transform to realize the projection of SE(n-1) onto SO(n). The general approach here is based upon preliminary works reported in [5, 19, 20].

Consider the space of $(n \times n)$ matrices as shown in Fig. 3. Let $[T]$ be a $(n \times n)$ homogeneous transform that represents an element of SE(n-1). Note that $[T]$ defines a point in R^{n^2}. $[A]$ is the desired element of SO(n) nearest $[T]$ when it lies in a direction orthogonal to the tangent plane of SO(n) at $[A]$.

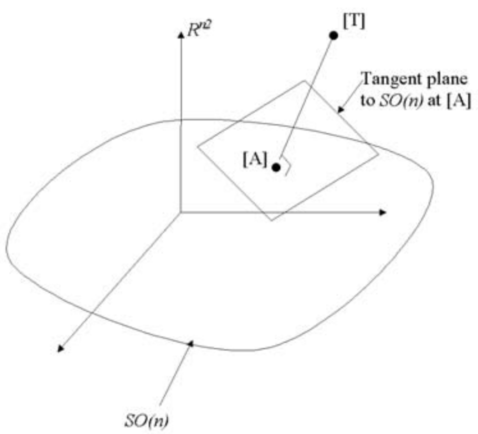

Figure 3. General Case: SE(n-1) ⇒ SO(n)

The following theorem, based upon related works by Hanson and Norris [13] provides the foundation for the projection,

Theorem 0.1. *Given any $(n \times n)$ matrix $[T]$ the closest element of SO(n) is given by: $[A] = [U][V]^T$ where $[T] = [U][diag(s_1, s_2, \ldots, s_n)][V]^T$ is the SVD of $[T]$.*

Shoemake and Duff [29] prove that matrix $[A]$ satisfies the following optimization problem: *Minimize:* $\|[A]-[T]\|_F^2$ subject to: $[A]^T[A]-[I]=[0]$, where $\|[A]-[T]\|_F^2 = \sum_{i,j}(a_{ij}-t_{ij})^2$ is used to denote the Frobenius norm. Since $[A]$ minimizes the Frobenius norm in R^{n^2} it is the element of SO(n) that lies in a direction orthogonal to the tangent plane of SO(n) at $[R]$. Hence, $[A]$ is the closest element of SO(n) to $[T]$. Moreover, for full rank matrices the SVD is well defined and unique. We now restate Th. 0.1 with respect to the desired SVD based projection of SE(n-1) onto SO(n).

Theorem 0.2. *For $[T] \in$ SE(n-1) and $[T] = [U][diag(s_1, s_2, \ldots, s_n)][V]^T$ if $[A] = [U][V]^T$ then $[A]$ is the unique element of SO(n) nearest $[T]$.*

Recall that $[T]$, the homogenous representation of SE(n), is full rank ([25]) and therefore $[A]$ exists, is well defined, and unique.

THE PD BASED PROJECTION

The polar decomposition (PD), though perhaps less known than the SVD, is quite powerful and actually provides the foundation for the SVD. The polar decomposition theorem of Cauchy states that "a non-singular matrix equals an orthogonal matrix either pre or post multiplied by a positive definite symmetric matrix", see [12]. With respect to our application, for $[T] \in$ SE(n-1) its PD is $[T] = [P][Q]$, where $[P]$ and $[Q]$ are $(n \times n)$ matrices such that $[P]$ is orthogonal and $[Q]$ is positive definite and symmetric. Recalling the properties of the SVD, the decomposition of $[T]$ yields $[U][\text{diag}(s_1, s_2, \ldots, s_{n-1})][V]^T$, where matrices $[U]$ and $[V]$ are orthogonal and matrix $[\text{diag}(s_1, s_2, \ldots, s_{n-1})]$ is positive definite and symmetric. Moreover, it is known that for full rank square matrices that the polar decomposition and the singular value decomposition are related by: $[P] = [U][V]^T$ and $[Q] = [V][\text{diag}(s_1, s_2, \ldots, s_{n-1})][V]^T$, [9]. Hence, for $[A] = [U][V]^T$ we have $[A]$ = $[P]$ and conclude that the polar decomposition yields the same element of SO(n). We now restate Th. 0.2 with respect to the desired PD based projection of SE(n-1) onto SO(n).

Theorem 0.3. *If $[T] \in$ SE(n-1) and $[T] = [P][Q]$ then $[P]$ is the unique element of SO(n) nearest $[T]$.*

COMPUTATIONAL ISSUES

Often, the evaluation of the singular value decomposition is implemented in code by computing the eigenvalues and eigenvectors of the matrix since the singular values are the positive square roots of the eigenvalues of $[T][T]^T$ and the columns of

$[U]$ and $[V]$ are the normed eigenvectors of $[T][T]^T$ and $[T]^T[T]$ respectively. However, we are computing the SVD of a homogeneous transform representing SE(n-1). The eigenvalue and eigenvectors of SE(2) and SE(3) are well known and should be exploited to facilitate the computations, see [25].

With regard to the PD, a simple and efficient iterative algorithm exists for its evaluation. Dubrulle [6] provides an algorithm that produces monotonic convergence in the Frobenius norm that "... generally delivers an IEEE double-precision solution in ~ 10 or fewer steps". A MatLab implementation of Dubrulle's algorithm is shown in Fig. 4.

```
function P=polar(T)
%
% initialization
%
P=T;
limit = (1 + eps) * sqrt(size(T,2));
T = inv(P');
g = sqrt(norm(T,'fro')/norm(P,'fro'));
P=0.5*(g*P+(1/g)*T);
f = norm(P,'fro');
pf = inf;
%
% iteration
%
while (f>limit) & (f<pf)
    pf = f;
    T = inv(P');
    g=sqrt(norm(T,'fro')/f);
    P=0.5*(g*P+(1/g)*T);
    f=norm(P,'fro');
end
return
```

Figure 4. Dubrulle's PD Algorithm: MatLab Implementation

THE CHARACTERISTIC LENGTH

In order resolve the unit disparity between translations and rotations we use a characteristic length to normalize the translational terms in the displacements. The characteristic length we chose to use, based upon the investigations reported in [8, 21], is $\mathrm{R} = \frac{24L}{\pi}$ where L is the maximum translational component in the set of displacements at hand. This characteristic length is the radius of the hypersphere that approximates the translational terms by angular displacements that are $\leq 7.5(\mathrm{deg})$. It was shown in [15] that this radius yields an effective balance between translational and rotational displacement terms for projection metrics.

Finally, it is important to recall that both the SVD and PD based projections of SE(n-1) onto SO(n) are coordinate frame and unit dependent. This is true for all metrics on spatial and planar displacements as no bi-invariant metric exists, see [2] and [23]. Note however that these mappings project SE(n-1) onto SO(n) and bi-invariant metrics do exist on SO(n).

ONE METRIC ON SO(n)

One useful and easily computed metric d on SO(n) follows. Given two elements $[A_1]$ and $[A_2]$ of SO(n) we can define a metric using the Frobenius norm as,

$$d = \|[I] - [A_2][A_1]^T\|_F. \tag{5}$$

It is straightforward to verify that this is a valid metric on SO(n), see [28].

CASE STUDY-1

Consider a planar displacement $(a, b, \alpha) = (0, 1, 0)$. Its corresponding element of SE(2) is $[T]$ and we compute its projection $[A]$ onto SO(3) using either technique presented here and yield:

$$[T] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \tag{6}$$

and

$$[A] = \begin{bmatrix} 1.0 & 0.0 & 0.0 \\ 0.0 & 0.9979 & 0.0653 \\ 0.0 & -0.0653 & 0.9979 \end{bmatrix} \tag{7}$$

where $\mathrm{R} = 7.6394$. It is illustrative to compute the axis of rotation $\mathbf{s} = [-1.0\ 0.0\ 0.0]^T$ and the angle of rotation $\eta = 3.7440(\mathrm{deg})$ associated with $[A]$, see Fig. 5. Moreover, the longitude, latitude, and roll angles of $[A]$ are: $\theta = 0.0$, $\phi = 3.7440(\mathrm{deg})$, and $\psi = 0.0$ as expected. Finally, from Eq. 5 we determine the magnitude of this displacement with respect to the identity to be $\|[T]\| = 0.0924$.

CASE STUDY-2

Consider another planar displacement $(a, b, \alpha) = (1, 1, 45)$. We proceed as in Case-1 and yield the following:

$$[T] = \begin{bmatrix} 0.7071 & -0.7071 & 1 \\ 0.7071 & 0.7071 & 1 \\ 0 & 0 & 1 \end{bmatrix} \tag{8}$$

and

$$[A] = \begin{bmatrix} 0.7041 & -0.7071 & 0.0652 \\ 0.7041 & 0.70714 & 0.0652 \\ -0.0922 & 0.0 & 0.9957 \end{bmatrix} \tag{9}$$

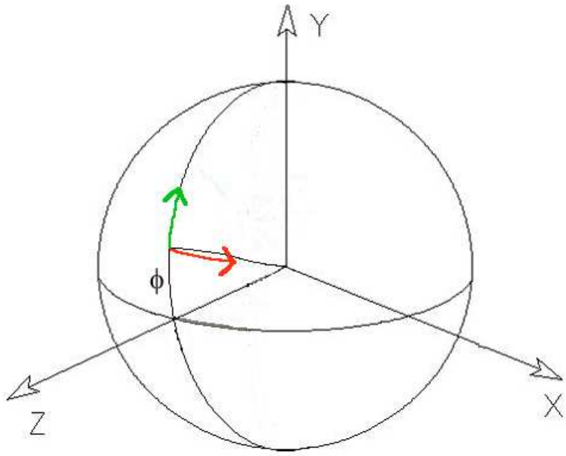

Figure 5. Planar Case: SE(2) ⇒ SO(3)

where R = 7.6394. Again, it is illustrative to compute the angle and axis of rotation $\eta = 45.29$(deg) and $\mathbf{s} = [-0.0459\ 0.1107\ 0.9928]^T$, see Fig. 1. Moreover, the longitude, latitude, and roll angles associated with $[A]$ are: $\theta = 3.75$, $\phi = 3.74$, and $\psi = 44.88$(deg). Finally, from Eq. 5 we have $\|[T]\| = 1.0891$.

CASE STUDY-3

Consider a spatial displacement $(d_x, d_y, d_z, \theta, \phi, \psi) = (1, 2, 3, 10, 30, 75)$. We proceed as above and yield the following:

$$[T] = \begin{bmatrix} 0.1710 & -0.9737 & 0.1540 & 1.0000 \\ 0.8365 & 0.2241 & 0.5000 & 2.0000 \\ -0.5206 & 0.0403 & 0.8529 & 3.0000 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (10)$$

$$[A] = \begin{bmatrix} 0.1710 & -0.9736 & 0.1495 & 0.0217 \\ 0.8364 & 0.2243 & 0.4982 & 0.0435 \\ -0.5208 & 0.0406 & 0.8502 & 0.0652 \\ -0.0061 & 0.0088 & -0.0806 & 0.9967 \end{bmatrix}, \quad (11)$$

where R = 22.9183 and from Eq. 5 we have $\|[T]\| = 1.8750$.

CONCLUSIONS

We have presented two projection metrics on SE(n). These metrics are based on projections of SE(n) onto SO(n+1) that utilize the singular value and polar decompositions of the homogeneous transform representations of SE(n). It was shown that both methods yield the same projection that determines the element of SO(n+1) nearest the given element of SE(n). Any bi-invariant metric on SO(n+1) may then be used to measure the *distance* between any two spatial displacements SE(n). The results are PD and SVD based projection metrics on SE(n). These metrics have applications in motion synthesis, robot calibration, motion interpolation, and hybrid robot control.

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