Synthesis of Part Orienting Devices for Spatial Assembly Tasks

P. Larochelle

Robotics & Spatial Systems Lab, Department of Mechanical and Aerospace Engineering, Florida Institute of Technology, Melbourne, FL 32901-6975, U.S.A., e-mail: pierrel@fit.edu

Abstract. A novel dimensional synthesis technique for solving the mixed exact and approximate motion synthesis problem for spherical RR kinematic chains is presented. The methodology uses an analytic representation of the spherical RR dyad's rigid body constraint equation in combination with classical geometric constructions for exact motion synthesis to yield designs that exactly reach two of the prescribed orientations while approximating the remaining. The result is a mixed exact and approximate motion dimensional synthesis technique that is applicable to spherical open and closed kinematic chains. Here, we specifically address the design of spherical RR open and 4R closed chains since they form the foundation of a new class of devices being developed called PODs or Part Orienting Devices. An example that demonstrates the synthesis technique is included.

Key words: spherical mechanisms, exact motion synthesis, approximate motion synthesis.

1 Introduction

As a product is assembled in an automated factory, both the product and its individual parts are picked up, reoriented and inserted into subassemblies or fixtures. For a complex product, the number of manipulations could run into the thousands. Parts are picked out of bins and placed into assemblies. Partial assemblies are rotated to allow additional parts to be added. Fasteners are inserted to hold everything together.

Typically, designers of assembly lines seek to keep the manipulations as simple as possible. Rotations about vertical or horizontal axes are preferred, often of 90 or 180 (deg). These tasks have a well established set of solutions. However, operations which involve a translation along and/or a rotation about an axis which is not vertical or horizontal is more challenging to the designer. Additional constraints on the trajectory of the object (e.g. obstacle avoidance, or part meshing) increase the difficulties. One solution is to use devices with a high number of degrees of freedom, such as industrial robots. Robots can perform the tasks, but at penalties in costs, setup time, and maintenance. A second solution is to use a cascading series

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of simple one degree of freedom devices. Creating this manipulation pipeline takes a longer design time and is often more art than science.

Part Orienting Devices (PODs) offer another alternative. The synthesis algorithm presented here is part of ongoing efforts directed at realizing the capability to design these devices for spatial assembly tasks. These low degree-of-freedom devices are capable of producing the necessary spatial reorientations often required in spatial assembly tasks. Hence, PODs provide an alternative for solving spatial assembly tasks that might otherwise require a robot or multiple single degree-of-freedom devices.

A well known result from screw theory is that moving an object from one spatial location to another does not require six degrees of freedom. In fact, such motions can be accomplished with a single degree of freedom twist about a screw axis. However, it is rare that this solution is practical due to the location of the screw axis within the workspace and the collisions and interferences between objects that result. PODs are low degree-of-freedom machines that are a compromise between the 6 or more degree of freedom industrial robot and the single degree of freedom twisting motion. Here, the focus is on utilizing the spherical 4R closed chain architecture to serve as the motion generator for a class of PODs to achieve two desired orientations exactly while approximating a set of guiding orientations that take the workpiece from one exact orientation to the other.

In related works [7] present the derivation of the constraint manifold for spherical RR dyads using the image space representation of displacements. Their work was an extension of the ideas presented in [8]. In [11] the design of spherical mechanisms to approximate spatial locations is presented. A robust synthesis algorithm for spherical motion generation was presented by Al-Widyan and Angeles [1]. More recently, [2] present the synthesis of spherical 4R mechanisms for 5 prescribed orientations. Related ongoing efforts at the University of Dayton to advance the design of PODs have been reported in [3, 6]. The methodology used here for performing the dimensional synthesis for mixed exact and approximate orientation rigid body guidance is based upon the works of Tsai and Roth [10] and McCarthy [5].

This paper proceeds as follows. First, the geometry and kinematics of the spherical RR dyad and the spherical 4R closed chains are reviewed. Next, the synthesis algorithm for solving the mixed exact and approximate motion synthesis problem for spherical RR kinematic chains is presented. Finally, an example POD design is presented; the synthesis of a spherical 4R closed chain to achieve two prescribed orientations exactly while approximating three guiding orientations.

2 Synthesis Algorithm

A spherical 4R closed chain may be viewed as the combination of two spherical RR dyads where each dyad consist of two R joints; one fixed and the other moving, see Figure 1. The approach taken here is to synthesize two dyads separately and then join their floating links to yield a kinematic closed chain. Let the fixed axis be specified by the vector $\mathbf{u}$ measured in the fixed reference frame F and let the moving axis be specified by $\mathbf{v}$ measured in the moving frame M. Moreover, let $\mathbf{l}$ define the moving axis $\mathbf{v}$ in the fixed frame F so that, $\mathbf{l} = [A]\mathbf{v}$ where $[A]$ is the element of SO(3) that defines M with respect to F. Because the link is rigid, the angle between the two axes of the dyad remains constant. This geometric constraint may be expressed analytically as,

$$\mathbf{u} \cdot \mathbf{l} = \mathbf{u} \cdot [A]\mathbf{v} = \cos\alpha. \tag{1}$$

This constraint equation is the foundation of the synthesis algorithm presented below. In order to solve the mixed exact and approximate synthesis problem we first solve the exact synthesis problem for 3 prescribed orientations.

2.1 *Exact Synthesis for Three Orientations*

First, we select the moving axis $\mathbf{v}$. Second, we write Eq. (1) for each of the desired orientations, $[A]_i$, $i = 1, 2, 3$. Finally, we subtract the first equation from the remaining two to arrive at a linear system of equations,

$$[P]\mathbf{u} = \mathbf{b}, \tag{2}$$

where

$$[P] = \begin{bmatrix} \mathbf{v}^T([A]_2 - [A]_1)^T \\ \mathbf{v}^T([A]_3 - [A]_1)^T \\ 0 \quad\quad 0 \quad\quad 1 \end{bmatrix},$$

$\mathbf{b} = [0\ 0\ 1]^T$, and $\mathbf{u}$ is the desired fixed axis. Note that we must solve Eq. (2) for each prescribed moving axis to find its corresponding fixed axis. Moreover, note that since we are using 3-vectors to define the axes when in fact they are simply directions that only require 2 coordinates, the last row of $[P]$ is chosen to yield the vector $\mathbf{u}$ that is the intersection of the fixed axis with the $z = 1$ plane. In the event that $[P]$ is rank deficient (i.e. when the fixed axis does not intersect the $z = 1$ plane) simply change the last row to any vector that does not lie in this plane (e.g. $[1\ 0\ 0]^T$).

2.2 *Mixed Synthesis Algorithm*

In the problem considered here we have 2 orientations to reach exactly and n orientations that serve to guide the body from one exact orientation to the other. First a desired moving axis $\mathbf{v}$ is selected. Next, we seek a corresponding fixed axis for the dyad. The fixed axis is found by solving n 3 orientation problems to yield a set of fixed axes $\mathbf{u}_i$, $i = 1, 2, \ldots, n$. The 3 orientation problems are derived from the 2 exact orientations along with 1 of the guiding orientations. Hence, there are n unique

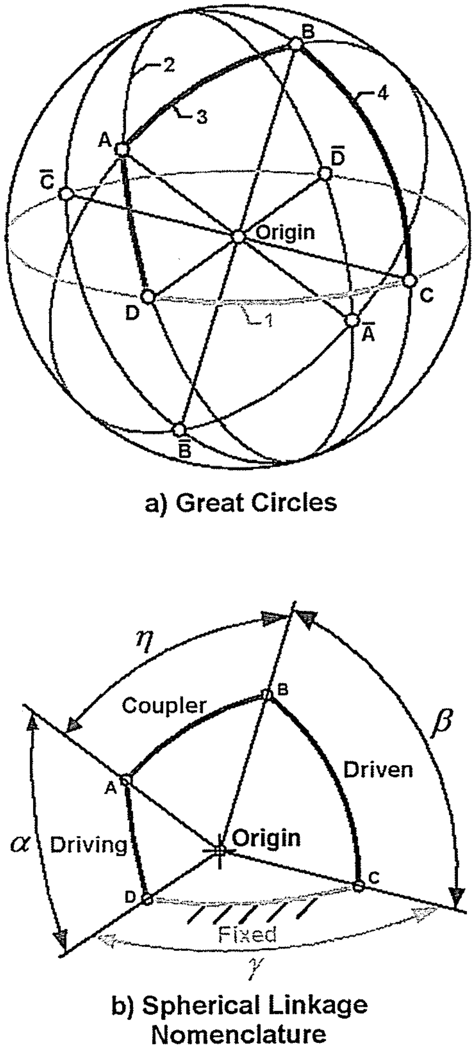

Fig. 1 Spherical 4R geometry and nomenclature.

3 orientation problems. We select the fixed axis $\mathbf{u}$ that is their normalized sum,

$$\mathbf{u} = \frac{\sum \mathbf{u}_i}{\| \sum \mathbf{u}_i \|}. \tag{3}$$

It is beneficial to discuss the geometry underlying this approach. Consider the synthesis of a spherical RR dyad for two exact orientations. Associated with the desired moving axis is a great circle that is defined by the set of all fixed axis that solve the problem. Now consider another orientation and one of the original 2 orientations.

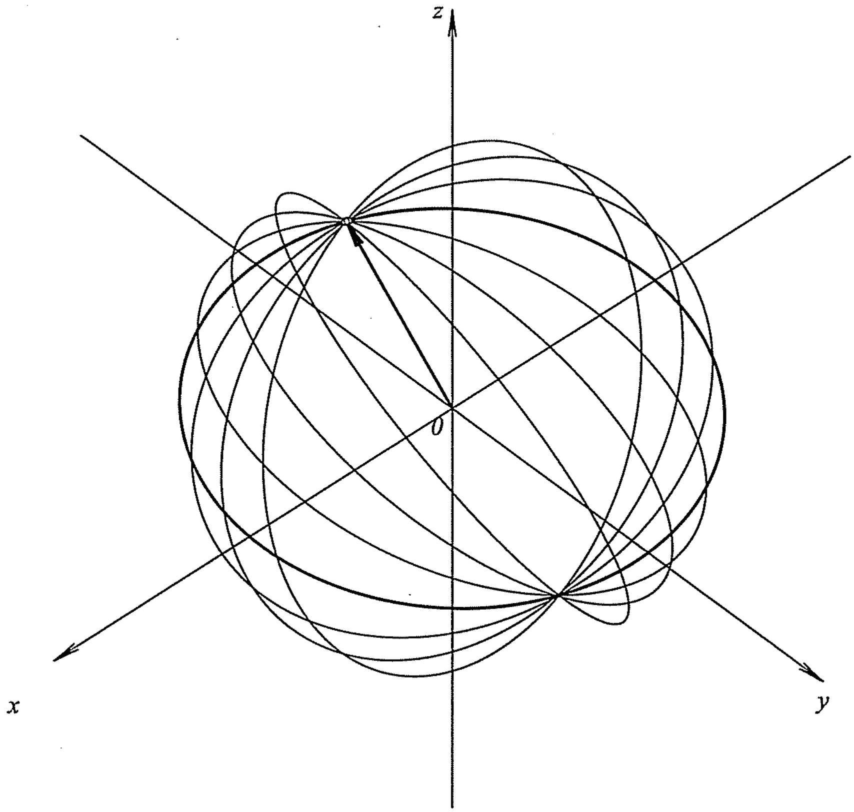

Fig. 2 The great circles associated with dyad #1.

For a desired moving axis there is again a great circle that represents all of the solution fixed axes. Fixed axes that guide a body through all 3 orientations must lie at the intersections of these two great circles. Generally, these great circles intersect in two points that define line; hence there is 1 unique fixed axis associated with 3 spherical orientations and a choice of moving axis. Finally, consider the exact 3 orientation problem. The desired fixed axis lies at the intersection of 3 great circles; the first associated with orientations 1 & 2, the second with 2 & 3, and the third with 1 & 3. By solving all of the 3 orientation problems that include the two exact orientations we guarantee that all resulting fixed axes lie on the great circle associated with these two orientations. Moreover, in Eq. (3) we utilize the fixed axis that lies on the great circle associated with the two exact orientations and that is nearest the great circles associated with the guiding orientations. Hence, the solution dyad will guide the part exactly through the two prescribed orientations and nearest the guiding orientations for the selected moving axis.

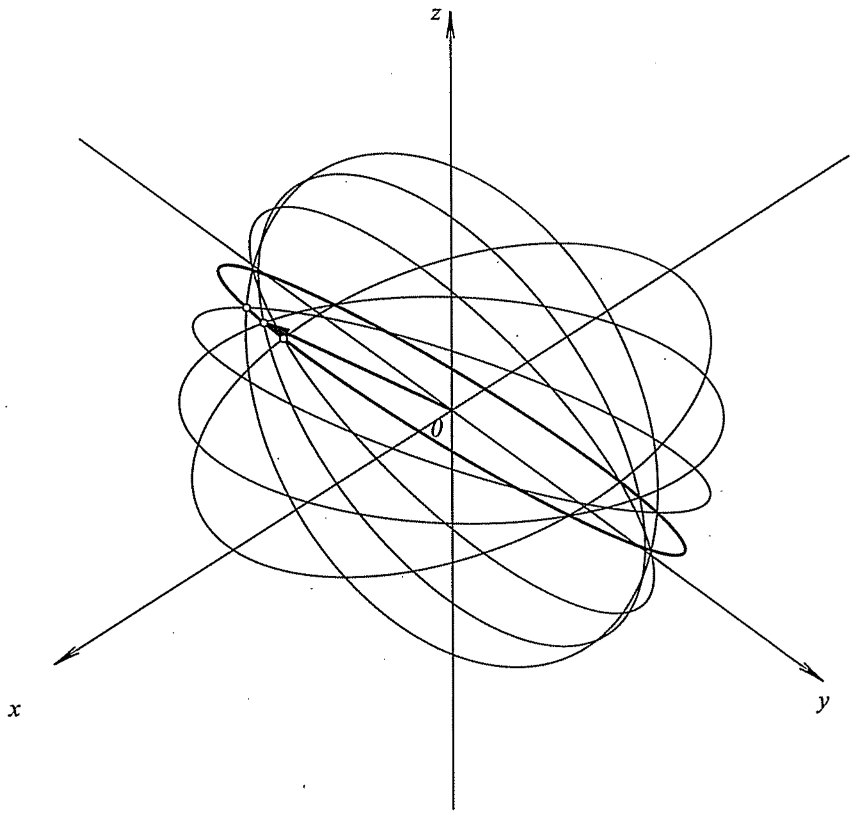

Fig. 3 The great circles associated with dyad #2.

3 Example

A common task in spatial assembly operations is the reorienting of a part by successive rotations of 90 (deg) about two orthogonal axes; the so called *90-90* problem. Here we employ the preceding methodology and design a POD to achieve the desired motion by synthesizing a spherical 4R mechanism for 5 orientations; 2 exact (the starting orientation and the final orientation after the 90-90 rotations) and 3 guiding orientations as defined in Table 1 where $[A] = [Rot_z(lng)][Rot_y(-lat)][Rot_x(rol)]$. In order to prescribe the size of the coupler link and to eliminate the need for any extension or attachment to connect the moving body to the coupler these moving axes were selected: $\mathbf{v}_1 = [1\ 1\ 1]^T$ and $\mathbf{v}_2 = [1\ 0\ 0]^T$ (see [4, 9]). The mixed synthesis algorithm yielded fixed axes: $\mathbf{u}_1 = [0.2745, -0.2745, 0.9216]^T$ and $\mathbf{u}_2 = [0.6780, -0.2839, 0.6780]^T$. The great circles that illustrate the application of the algorithm to determine $\mathbf{u}_1$ are shown in Figure 2 and those associated with $\mathbf{u}_2$ are shown in Figure 3. The great circle as-

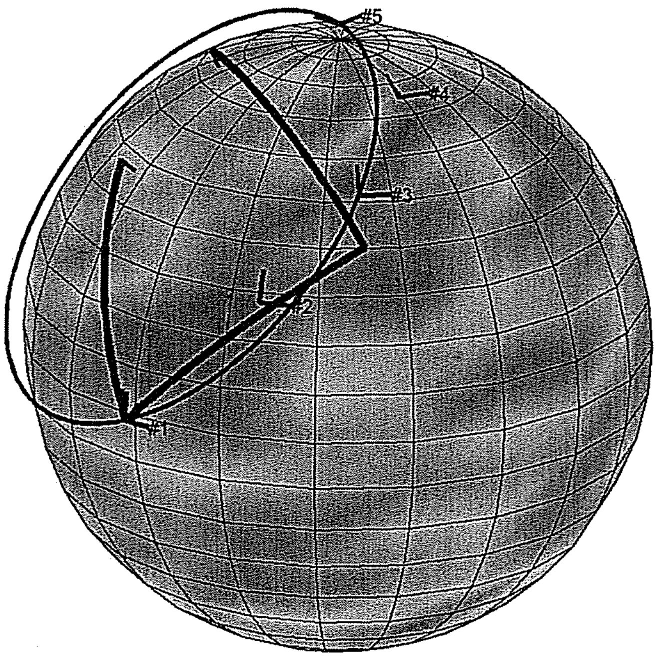

Fig. 4 The POD shown in orientation #1.

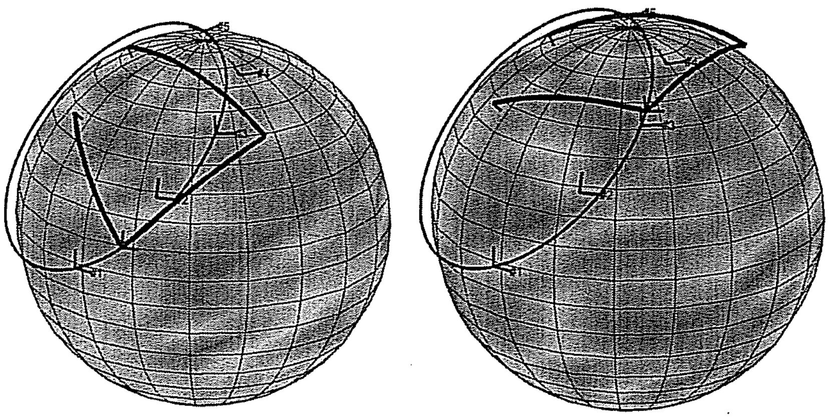

Fig. 5 Moving from orientation #1 to #2 (left) and from #3 to #4 (right).

sociated with the 2 exact orientations is thicker and the 3 fixed axes associated with orientations 1-2-3, 1-3-5, and 1-4-5 are indicated by ∘ symbols on the great circle. Recall that these axes are used in Eq. (3) to determine $\mathbf{u}_i$. The resulting POD is a Grashof double-crank spherical four-bar mechanism that does not suffer from circuit, branch, or order defects. Its link lengths are: $\alpha = 57.8512$, $\eta = 54.7321$,

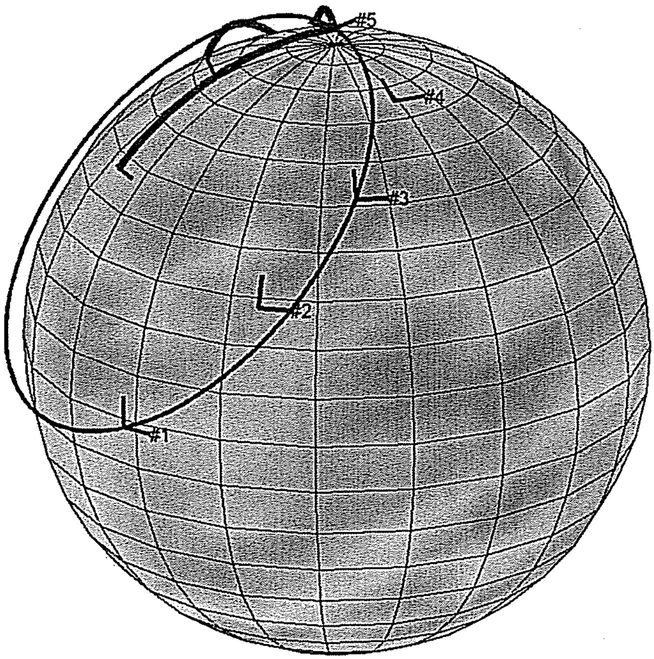

Fig. 6 The POD shown in orientation #5.

Table 1 Five prescribed orientations.

#	Longitude	Latitude	Roll	Motion Type
1	0.00	0.00	0.00	exact
2	25.00	25.00	0.00	approximate
3	45.00	45.00	0.00	approximate
3	65.00	65.00	0.00	approximate
5	90.00	90.00	0.00	exact

$\beta = 47.3688$, and $\gamma = 27.2574$ and the moving body is attached at 135 (deg) to the coupler at the driven moving axis. The solution POD is shown in Figures 4–6.

4 Conclusions

A novel dimensional synthesis technique for solving the mixed exact and approximate motion synthesis problem for spherical RR open and 4R closed kinematic chains has been presented. The methodology uses an analytic representation of the spherical RR dyad's rigid body constraint equation in combination with classical geometric constructions for exact motion synthesis to yield designs that exactly reach two of the prescribed orientations while approximating the remaining guiding orientations. Such part orienting tasks are common in automated assembly systems. Here, we specifically address the design of spherical RR open and 4R closed chains since they serve as the motion generators for a class of PODs, or part orienting devices, that are being developed.

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