Proceedings of the ASME 2010 International Design Engineering Technical Conferences & Computers and Information in Engineering Conference
IDETC/CIE 2010
August 15-18, 2010, Montreal, Quebec, Canada

DETC2010-28072

A HOOP METHOD FOR ORIENTATION ORDER ANALYSIS OF SPHERICAL MECHANISMS

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ABSTRACT

In this paper we present a novel methodology for orientation order analysis of spherical RR dyads. The methodology is a spherical generalization of the recent works of Myszka, Murray, and Schmiedeler for assessing position order of planar RR dyads. The objective of the methodology is to determine if a prescribed fixed axis location for a spherical RR dyad will result in the dyad guiding a moving rigid-body through a set of finitely separated spherical orientations in the desired order (e.g. 1, 2, 3, 4, etc.).

First, the prior works on the order analysis of planar RR dyads via the propeller method are briefly reviewed. Next, the planar propeller methodology of Myszka, Murray, and Schmiedeler is extended to yield a spherical hoop methodology. The result is a useful tool to determine if a given spherical RR dyad will guide a moving body through a set of prescribed orientations in the desired order. Finally, we demonstrate the utility of the hoop method for order analysis of spherical RR dyads in three case studies.

Keywords

Hoop Method, Spherical RR Dyad, Spherical Mechanism, and Order Defect.

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INTRODUCTION

The objective of this effort is to devise a methodology to determine whether a given prescribed fixed axis of a spherical RR dyad guides a rigid body through n desired spherical orientations in the desired order (1, 2, 3, 4 ...). The motivation derives from commonly encountered kinematic dimensional synthesis challenges. A common mechanism or linkage design objective is the dimensional synthesis to solve rigid-body guidance tasks [1]; also known as motion generation tasks [2]. In such cases the objective is to determine the geometric parameters that define the mechanism such that the mechanism guides a moving body through a sequence of finitely separated locations in a desired order. Often kinematic synthesis algorithms for motion generation tasks yield large sets of candidate solutions that must be analyzed for motion defects; see *SYNTHETICA* by Su et. al. [3], *LINCAGES* by Erdman [4], *SPHINX* by Larochelle et. al. [5], and *SPHINX-PC* by Ruth and McCarthy [6]. Common motion defects encountered when using such software tools include order, circuit, and branch defects [7]. In a one degree of freedom mechanism, such as planar and spherical four-bars, the mechanism suffers from an order defect when unidirectional motion of the input or driving link does not guide the moving body through the prescribed locations in the desired order. Here we present a novel hoop method for analyzing spherical RR dyads and four-bar mechanisms for order defects. The hoop method [8] is derived from the propeller method; an earlier work done on the order analysis of planar mechanisms by Myszka, Murray, and Schmiedler [9, 10].

Subsequent to Chase and Mirth's work on defining and classifying motion defects [7], Balli and Chand [11] provided a comprehensive overview of linkage defects i.e. order, branch, and circuit defects. Filemon [12] presents graphical construction techniques for four position synthesis that identify regions of the design solution space that are free from order and circuit defects. Moreover, Filemon showed that in the case of four-bar mechanisms order defects are directly related to the driving (or input) dyad and are independent of the driven (or output) dyad. Hence the spherical RR dyad order analysis methodology presented here is directly applicable to the order analysis of spherical four-bar mechanisms. Additional works examining the order analysis for both planar and spherical mechanisms can be found in [9, 10, 13–19].

Recently Myszka, Murray, and Schmiedeler [9, 10] presented a new approach to determine whether a planar RR dyad will guide a moving body through a set of finitely separated positions in the desired order. Their methodology, the propeller method, utilizes relative crank angles and relative displacement poles to check the order of finitely separated planar locations associated with a prescribed fixed pivot **G** of a planar RR dyad. Here their theoretical work on order analysis of planar mechanisms is reviewed and extended to spherical mechanisms and we introduce the hoop method. For a given set of finitely separated spherical orientations and a spherical RR dyad with fixed axis **G** the hoop method utilizes the relative displacement axes of rotation to analyze the dyad for order defects.

THE PROPELLER METHOD

A kinematically elegant and geometrically intuitive approach for the order analysis of planar mechanisms, entitled the propeller method, was recently presented by Myszka, Murray, and Schmiedeler [9, 10], see Figure 1. First we summarize the implementation of the propeller method and then we review the derivation of the methodology.

A planar RR dyad consists of three bodies (the ground, link, and moving body) connected serially by two revolute joints. For a given planar RR dyad with fixed pivot **G** and a set of n desired locations for the moving body to be guided through the propeller method is implemented as follows. First an infinitely long line, referred to as the propeller, is defined. The propeller initially passes through the fixed pivot **G** and the relative displacement pole $\mathbf{P_{12}}$ and is then rotated in a counter-clockwise or clockwise direction (corresponding to the direction of motion of the dyad) for π radians. The order in which the dyad guides the moving body through the desired locations is established by tracking the sequence in which the propeller intersects the relative displacement poles associated with the n desired locations. Myszka, Murray, and Schmiedeler [10] show that for the dyad to guide the moving body through four

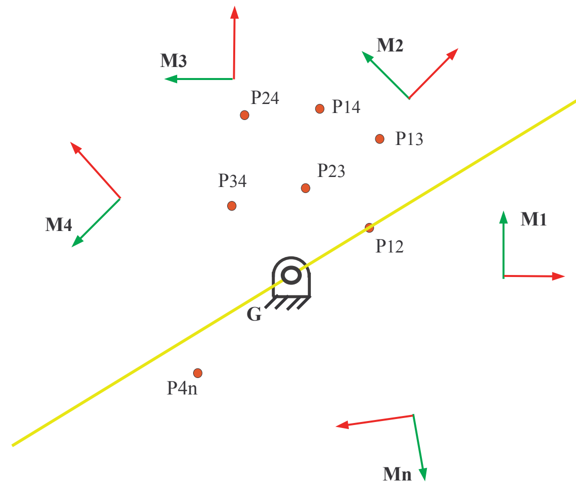

Figure 1. PROPELLER METHOD

locations in numerical sequence that the propeller must intersect the relative displacement poles in the following sequence $\mathbf{P_{13}} \Rightarrow \mathbf{P_{14}} \Rightarrow \mathbf{P_{24}}$. Moreover, for five locations they show that the sequence must be $\mathbf{P_{13}} \Rightarrow \mathbf{P_{14}} \Rightarrow \mathbf{P_{15}} \Rightarrow \mathbf{P_{25}}$. Note that in Figure 1 the propeller is shown in yellow and in its initial orientation. The general result is stated in Myszka, Murray, and Schmiedeler's propeller theorem rewritten here as follows.

Theorem 1. *A planar RR dyad will guide the moving body through locations* 1 *to* n*, in numerical sequence, if when the propeller is rotated* π *degrees about the fixed pivot* **G** *(in the same direction that the dyad is driven in) it intersects the relative displacement poles in the order* $\mathbf{P_{13}} \Rightarrow \mathbf{P_{14}} \Rightarrow \ldots \Rightarrow \mathbf{P_{1n}} \Rightarrow \mathbf{P_{2n}}$.

Corollary: *If the propeller does not intersect the poles in the order* $\mathbf{P_{13}} \Rightarrow \mathbf{P_{14}} \Rightarrow \ldots \Rightarrow \mathbf{P_{1n}} \Rightarrow \mathbf{P_{2n}}$ *then the dyad will not guide the moving body through the locations in their numerical sequence.*

The propeller theorem and methodology are derived from fundamental properties of crank rotation angles and relative poles. For a dyad to guide a body through the locations in the desired order 1, 2, 3, 4, ... the relative crank angles β_{ij} must monotonically increase where β_{ij} is the change in crank angle from location i to location j, see Figure 2. Note that without loss of generality we regard counter-clockwise crank rotations to be positive so that $0 \leq \beta_{ij} \leq 2\pi$, $\forall i, j < n$ and $i < j$. Therefore, a dyad does not exhibit an order defect if

$$0 < \beta_{12} < \beta_{13} < \cdots < \beta_{1n} < 2\pi \qquad (1)$$

Next, the method utilizes McCarthy's center-point theorem [20] to express the relationships between the fixed pivot **G**,

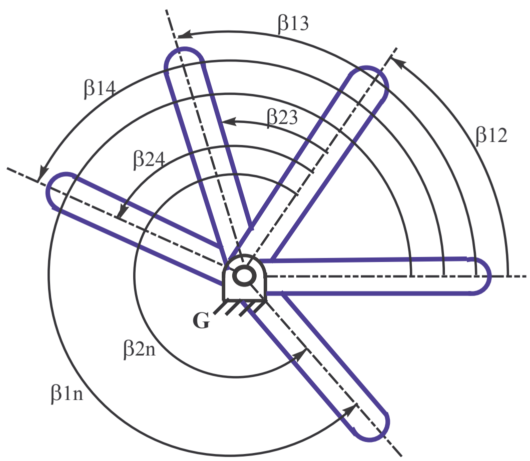

Figure 2. PLANAR RR DYAD AND CRANK ANGLES

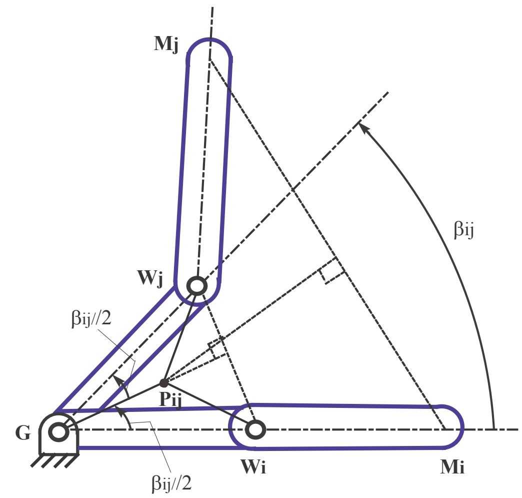

Figure 3. THE CENTER-POINT THEOREM (Reproduced from [20])

the relative pole $\mathbf{P_{ij}}$, and the relative crank angle β_{ij}. The center-point theorem states that $\angle \mathbf{P_{ij} G P_{jk}} = \frac{\beta_{ik}}{2}$ or $\frac{\beta_{ik}}{2} + \pi$, see Figures 3 and 4. Using the center-point theorem n necessary and sufficient conditions on the relative pole locations are then derived.

$$\begin{array}{l} 0 < \angle \overline{\mathbf{GP_{12}}}\,\overline{\mathbf{GP_{13}}} < \angle \overline{\mathbf{GP_{12}}}\,\overline{\mathbf{GP_{14}}} < \cdots < \angle \overline{\mathbf{GP_{12}}}\,\overline{\mathbf{GP_{1n}}} < \pi \\ 0 < \angle \overline{\mathbf{GP_{12}}}\,\overline{\mathbf{GP_{23}}} < \angle \overline{\mathbf{GP_{12}}}\,\overline{\mathbf{GP_{24}}} < \cdots < \angle \overline{\mathbf{GP_{12}}}\,\overline{\mathbf{GP_{2n}}} < \pi \\ 0 < \angle \overline{\mathbf{GP_{13}}}\,\overline{\mathbf{GP_{23}}} < \angle \overline{\mathbf{GP_{13}}}\,\overline{\mathbf{GP_{34}}} < \cdots < \angle \overline{\mathbf{GP_{13}}}\,\overline{\mathbf{GP_{3n}}} < \pi \\ \qquad \vdots \qquad\qquad\qquad\qquad \vdots \\ 0 < \angle \overline{\mathbf{GP_{1n}}}\,\overline{\mathbf{GP_{2n}}} < \angle \overline{\mathbf{GP_{1n}}}\,\overline{\mathbf{GP_{3n}}} < \cdots < \angle \overline{\mathbf{GP_{1n}}}\,\overline{\mathbf{GP_{n-1,n}}} < \pi \end{array} \tag{2}$$

Equations 2 state that a dyad will guide a body through the n locations in numerical order if for all k, starting with $\mathbf{P_{ik}}$, the propeller intersects every pole involving location k in ascending order.

SPHERICAL FOUR-BAR MECHANISMS

A four-bar mechanism consists of four links connected by four revolute joints. The links of the spherical four-bar mechanism can be represented by arcs of great circles. The links are connected to each other by revolute joints whose four axes intersect in a unique point; the center of the sphere of motion or design sphere. The links of the mechanism undergo spherical motion because the four joint axes intersect in a unique point. A rigid-body undergoing spherical motion has three degrees of freedom; rotations about three mutually orthogonal axes passing through the center of the sphere. These three rotation angles

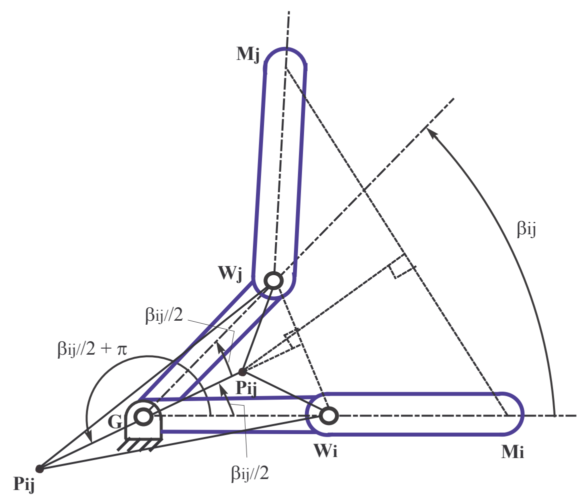

Figure 4. THE CENTER-POINT THEOREM WITH $\mathbf{P_{12}}$ ON THE OPPOSITE SIDE OF $\mathbf{G}$ (Reproduced from [20])

define the orientation of the body with respect to the fixed coordinate frame with origin at the center of the design sphere. Finally, here the longitude, latitude and roll angles [21] are used to define orientations of the moving body.

A spherical four-bar mechanism as shown in Figure 5 can be viewed as a combination of two spherical RR dyads, a driving dyad and a driven dyad. The driving fixed joint axis **O** and the driving moving axis **A** constitute the driving dyad whereas the driven fixed joint axis **C** and the driven moving axis **B** comprise the driven dyad. The link lengths of the mechanism are the driving link α, the driven link β, the coupler η and the ground or fixed link γ respectively. Often the two RR dyads

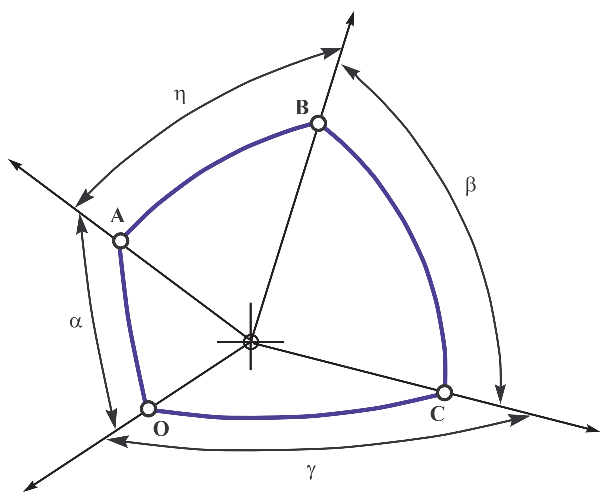

Figure 5. SPHERICAL FOUR-BAR MECHANISM

are synthesized separately and then their floating links are joined to form a coupler to yield a closed spherical kinematic chain. Recall that Filemon [12] showed that the order defects of a four-bar mechanism are independent of the driven dyad hence only the driving dyad of the spherical four-bar mechanism need be considered when performing an order analysis.

THE HOOP METHOD

Here we extend the planar propeller method of Myszka, Murray, and Schmiedeler [9, 10] to spherical RR dyads. The hoop method is a novel methodology for orientation order analysis for spherical RR dyads performing rigid-body guidance. The methodology utilizes relative crank angles and relative displacement axes of rotation to check the order of finitely separated planar locations associated with a prescribed fixed axis **G** of a spherical RR dyad.

The derivation of the hoop method is a direct and straightforward spherical generalization of the derivation of the propeller method. The major steps in the derivation will be summarized here and the spherical nuances will be discussed in detail. Just as in the case of the planar propeller theorem and methodology, the spherical hoop method is derived from fundamental properties of crank rotation angles and relative axes of rotation. Moreover, McCarthy's spherical generalization of the center-point theorem, called the center-axis theorem [20], is utilized to yield necessary and sufficient conditions on the locations of the axes of rotation.

The planar geometric entities in the propeller method and their spherical analogs in the hoop method are summarized in Table 1. A point in the plane, e.g. the fixed axis **G**, corresponds in the spherical case to a line along the revolute joint's axis that passes through the center of the design sphere. Note that this line intersects the design sphere in two points, see Figure 6. In the hoop method this ambiguity is resolved by utilizing the intersection that correlates to the physical location of the joint that connects the fixed and moving links.

We proceed as in the case of planar RR dyad order analysis and note that Equation 1 applies to spherical RR dyads as well. Next, the center-axis theorem of McCarthy [20] expresses the geometric relationships between the fixed axis **G**, the relative rotation axis $\mathbf{S_{ij}}$ and the relative crank angle β_{ij} for a spherical RR dyad that guides a moving body through orientations $\mathbf{M_i}$, $\mathbf{M_j}$, and $\mathbf{M_k}$. The center-axis theorem states that $\angle \mathbf{S_{ij} G S_{jk}} = \frac{\beta_{ik}}{2}$ or $\frac{\beta_{ik}}{2} + \pi$, see Figure 7. Using the center-axis theorem n necessary and sufficient conditions on the locations of the relative axes of rotation are derived per the planar case presented in [10].

$$
\begin{aligned}
0 < \quad & \angle \overline{\mathbf{GS_{12}}}\,\overline{\mathbf{GS_{13}}} < \angle \overline{\mathbf{GS_{12}}}\,\overline{\mathbf{GS_{14}}} < \cdots < \angle \overline{\mathbf{GS_{12}}}\,\overline{\mathbf{GS_{1n}}} < \pi \\
0 < \quad & \angle \overline{\mathbf{GS_{12}}}\,\overline{\mathbf{GS_{23}}} < \angle \overline{\mathbf{GS_{12}}}\,\overline{\mathbf{GS_{24}}} < \cdots < \angle \overline{\mathbf{GS_{12}}}\,\overline{\mathbf{GS_{2n}}} < \pi \\
0 < \quad & \angle \overline{\mathbf{GS_{13}}}\,\overline{\mathbf{GS_{23}}} < \angle \overline{\mathbf{GS_{13}}}\,\overline{\mathbf{GS_{34}}} < \cdots < \angle \overline{\mathbf{GS_{13}}}\,\overline{\mathbf{GS_{3n}}} < \pi \\
\vdots \quad & \qquad\qquad\qquad\qquad \vdots \\
0 < & \angle \overline{\mathbf{GS_{1n}}}\,\overline{\mathbf{GS_{2n}}} < \angle \overline{\mathbf{GS_{1n}}}\,\overline{\mathbf{GS_{3n}}} < \cdots < \angle \overline{\mathbf{GS_{1n}}}\,\overline{\mathbf{GS_{n\text{-}1,n}}} < \pi
\end{aligned} \tag{3}
$$

Equations 3 state that a spherical RR dyad will guide a body through the n orientations in numerical order if for all k, starting with $\mathbf{S_{ik}}$, the hoop intersects every relative rotation axis involving orientation k in ascending order. This result is summarized in the following hoop theorem.

Theorem 2. *A spherical RR dyad will guide the moving body through orientations* 1 *to n, in numerical sequence, if when the hoop is rotated* π *degrees about the fixed axis* **G** *(in the same direction that the dyad is driven in) it intersects the relative axes of rotation in the order* $\boldsymbol{S_{13}} \Rightarrow \boldsymbol{S_{14}} \Rightarrow \ldots \Rightarrow \boldsymbol{S_{1n}} \Rightarrow \boldsymbol{S_{2n}}$.
Corollary: *If the hoop does not intersect the rotation axes in the order* $\boldsymbol{S_{13}} \Rightarrow \boldsymbol{S_{14}} \Rightarrow \ldots \Rightarrow \boldsymbol{S_{1n}} \Rightarrow \boldsymbol{S_{2n}}$ *then the dyad will not guide the moving body through the orientations in their numerical sequence.*

For a given spherical RR dyad with fixed axis **G** and a set of n desired orientations the hoop method is implemented as follows. First a great circle on the design sphere, referred to as the hoop, is defined. The hoop initially passes through the fixed axis **G** and the relative rotation axis $\mathbf{S_{12}}$ and is then rotated in a counter-clockwise or clockwise direction (corresponding to the direction of motion of the physical dyad) for π radians. The order in which the dyad guides the moving body through the desired orientations is established by tracking the sequence in which the hoop intersects the relative axes of rotation associated with the n desired orientations. Note that the hoop will intersect the relative

PLANAR PROPELLER METHOD	SPHERICAL HOOP METHOD
Fixed Pivot **G** (point)	Fixed Axis **G** (line)
Relative Pole $\mathbf{P_{12}}$ (point)	Relative Axis of Rotation $\mathbf{S_{12}}$ (line)
Propeller (line)	Hoop (great circle)

Table 1. PLANAR AND SPHERICAL CORRESPONDENCES

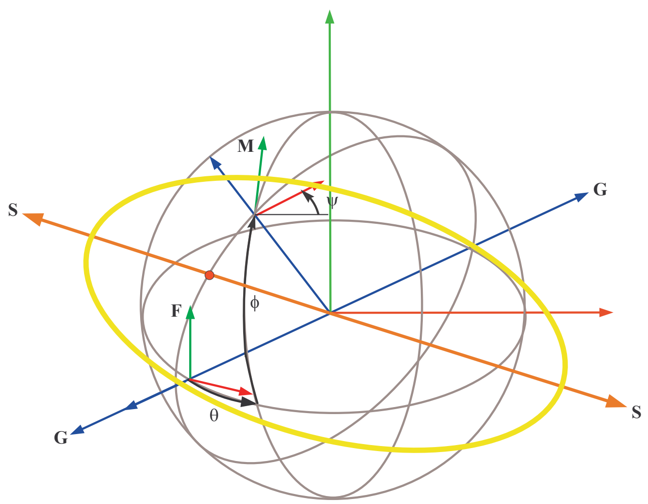

Figure 6. THE HOOP, FIXED AXIS $\mathbf{G}$, ROTATION AXIS $\mathbf{S}$, AND THE DESIGN SPHERE

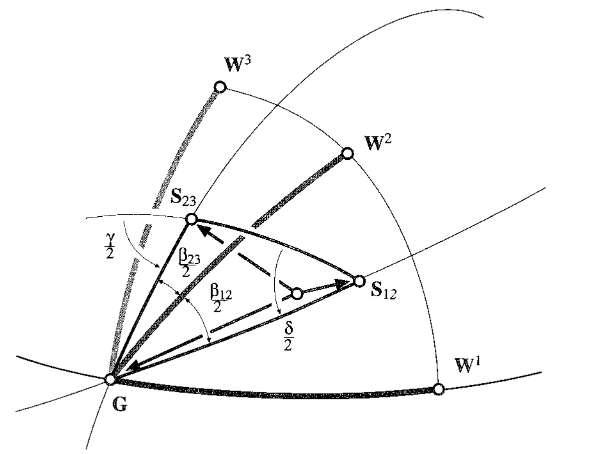

Figure 8.5. The center axis $\mathbf{G}$ views the relative rotation axes $\mathbf{S}_{12}$ and $\mathbf{S}_{23}$ in the angle $\beta_{13}/2 = \beta_{12}/2 + \beta_{23}/2$.

Figure 7. THE CENTER-AXIS THEOREM (Figure from [20])

rotation axes on both sides of the design sphere simultaneously. Furthermore, for the dyad to guide the moving body through four orientations in numerical sequence the hoop must intersect the relative rotation axes in the sequence $\mathbf{S_{13}} \Rightarrow \mathbf{S_{14}} \Rightarrow \mathbf{S_{24}}$. Moreover, for five orientations the sequence must be $\mathbf{S_{13}} \Rightarrow \mathbf{S_{14}} \Rightarrow \mathbf{S_{15}} \Rightarrow \mathbf{S_{25}}$. Note that in Figures 10, 12, and 13 the hoop is shown in yellow and in its initial orientation.

HOOP METHOD FLOWCHART

Given a spherical RR dyad that guides a rigid body through n orientations the task is to determine if the dyad guides the body through the orientations in the desired order (i.e. 1, 2, 3, 4 ....) or not. First, determine the $\binom{n}{2}$ (i.e. n choose 2) relative rotation axes associated with the n orientations. Next, generate the design sphere with fixed axis **G**, the relative rotation axes, and the hoop in its initial orientation. Proceed by rotating the hoop about **G** in the same direction that the dyad is to be actuated and record the sequence in which the hoop intersects the relative rotation axes. Finally the hoop theorem is used to analyze the sequence to determine if the orientations will be reached in the desired order. See Figure 8 for a flow-chart representation of the hoop methodology.

CASE STUDY #1

Consider five orientations defined by the four euler parameters x_1, x_2, x_3, and x_4 listed in Table 2 and the fixed axis **G** = $(0.054261, -0.996977, 0.055603)$ of a spherical RR dyad that solves the motion generation problem presented by Brunnthaler, Schröcker, and Husty [22]. In Figure 9 the five orientations $\mathbf{M_1}$, $\mathbf{M_2}$, $\mathbf{M_3}$, $\mathbf{M_4}$, $\mathbf{M_5}$ and the fixed axis **G** are shown on the design sphere. The next step is to determine the 10 relative rotation axes **S** associated with the spherical orientations. The hoop, a great circle on the design sphere passing through **G** and $\mathbf{S_{12}}$ is generated, see Figure 10. Next, as the hoop is rotated π radians in the counter-clockwise direction about **G** the sequence in which it intersects the relative rotation axes is noted. As the hoop is rotated in counter-clockwise turn of π radians about the fixed axis **G**, starting from the rotation axis $\mathbf{S_{12}}$, it intersects the remaining rotation axes in the order $\mathbf{S_{23}} \Rightarrow \mathbf{S_{45}} \Rightarrow \mathbf{S_{25}} \Rightarrow \mathbf{S_{13}} \Rightarrow \mathbf{S_{24}} \Rightarrow \mathbf{S_{15}} \Rightarrow \mathbf{S_{14}} \Rightarrow \mathbf{S_{35}} \Rightarrow \mathbf{S_{34}}$. The order indicates, per the hoop theorem, that this RR dyad will not guide the moving body

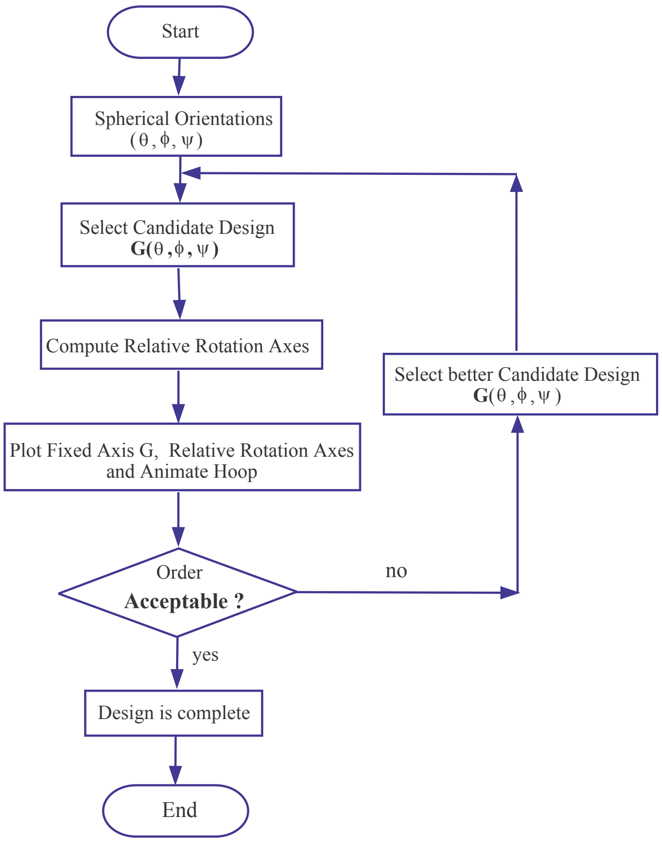

Figure 8. FLOW CHART FOR THE ORDER ANALYSIS OF SPHERICAL RR DYADS

Table 2. CASE STUDIES 1 and 2- FIVE ORIENTATIONS

No.	x_0	x_1	x_2	x_3
M_1	1.0	0.0	0.0	0.0
M_2	0.37721	0.82336	0.38967	0.16722
M_3	0.0078934	0.041131	0.085164	-0.99549
M_4	0.039457	0.77456	-0.60494	-0.18041
M_5	-0.30301	-0.36492	0.85697	0.20157

through the spherical orientations in their numerical sequence. Hence this dyad suffers from an order defect.

CASE STUDY #2

Here, we again consider the same five orientation problem with another solution spherical dyad presented in [22]. The fixed axis $\mathbf{G} = (-0.349442, -0.144163, 0.925801)$ of another spherical RR dyad that solves the motion generation problem

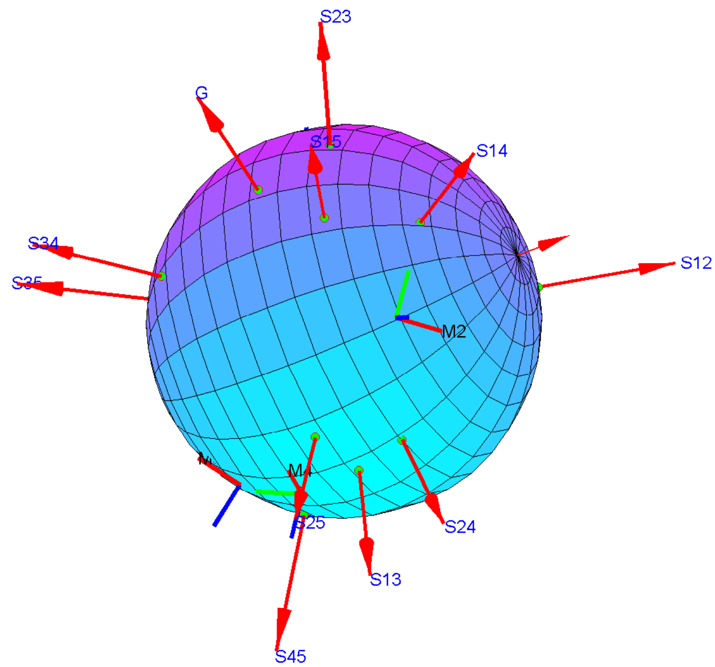

Figure 9. FIVE SPHERICAL ORIENTATIONS WITH THE FIXED AXIS **G**

is now analyzed. We proceed as before and apply the hoop methodology. In Figure 11 the five orientations, the fixed axis **G**, and the relative rotation axes are shown on the design sphere. As the hoop is rotated clockwise turn of π radians about **G**, starting from $\mathbf{S_{12}}$ (see Figure 12), it intersects the remaining rotation axes in the order $\mathbf{S_{35}} \Rightarrow \mathbf{S_{13}} \Rightarrow \mathbf{S_{45}} \Rightarrow \mathbf{S_{14}} \Rightarrow \mathbf{S_{23}} \Rightarrow \mathbf{S_{15}} \Rightarrow \mathbf{S_{24}} \Rightarrow \mathbf{S_{34}} \Rightarrow \mathbf{S_{25}}$. The order indicates, per the hoop theorem, that this RR dyad will guide the moving body through the spherical orientations in their numerical sequence. Note that the hoop theorem requires that the rotation axes $\mathbf{S_{13}}, \mathbf{S_{14}}, \mathbf{S_{15}}, \mathbf{S_{25}}$ must be intersected in the order: $\mathbf{S_{13}} \Rightarrow \mathbf{S_{14}} \Rightarrow \mathbf{S_{15}} \Rightarrow \mathbf{S_{25}}$. That is to say the intersections with the other rotation axes are irrelevant when applying the hoop theorem. Hence this dyad does not suffer from an order defect.

CASE STUDY #3

Consider the four orientations $\mathbf{M_1}$, $\mathbf{M_2}$, $\mathbf{M_3}$, $\mathbf{M_4}$ shown in Table 3 and the fixed axis $\mathbf{G}=(0.0742, 0.7117, 0.6986)$ of a spherical RR dyad that solves this motion generation problem. Here the actuated crank rotation is in the clockwise direction so the hoop is also rotated in a clockwise turn of π radians about the fixed axis **G**, see Figure 13. The hoop intersects the relative rotation axes in the sequence $\mathbf{S_{23}} \Rightarrow \mathbf{S_{13}} \Rightarrow \mathbf{S_{34}} \Rightarrow \mathbf{S_{14}} \Rightarrow \mathbf{S_{24}}$.

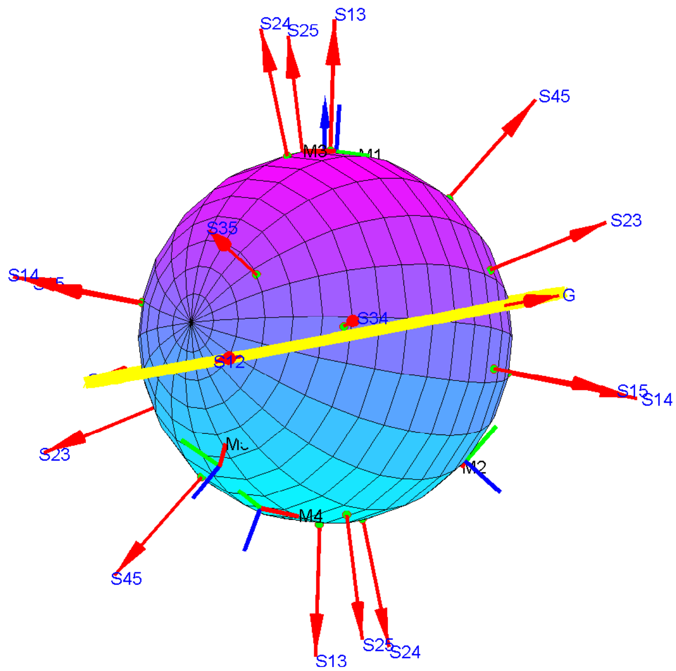

Figure 10. THE HOOP THROUGH FIXED AXIS **G** AND THE ROTATION AXIS $\mathbf{S}_{12}$

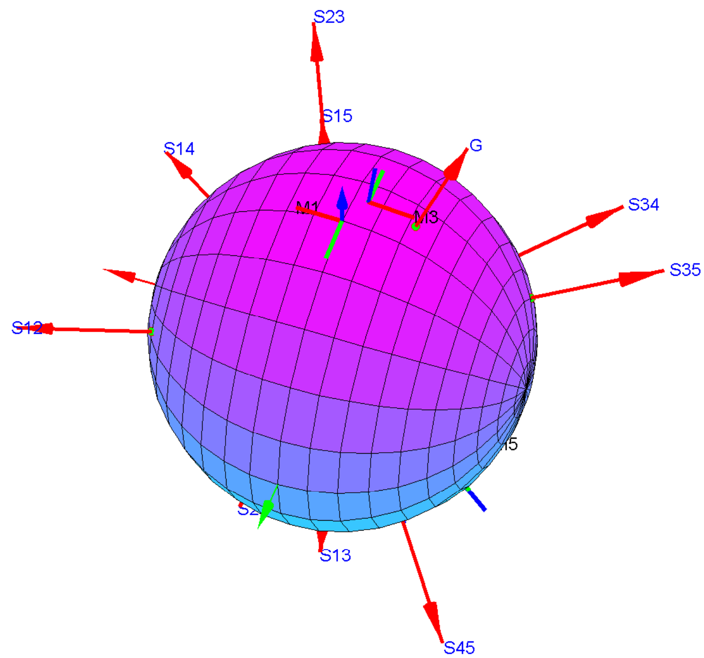

Figure 11. FIVE ORIENTATIONS WITH THE FIXED AXIS **G**

Table 3. CASE STUDY 3- FOUR ORIENTATIONS

No.	Longitude	Latitude	Roll
M_1	-80.0	80.0	40.0
M_2	20.0	60.0	124.0
M_3	60.0	-80.0	45.0
M_4	-80.0	80.0	5.0

Using the hoop theorem we conclude that this spherical RR dyad does not suffer from an order defect.

CONCLUSIONS

A novel methodology for orientation order analysis of spherical RR dyads has been presented. The methodology is a spherical generalization of the recent works of Myszka, Murray, and Schmiedeler for assessing position order of planar RR dyads.

The prior works on the order analysis of planar RR dyads via the propeller method were briefly reviewed. Next, the planar propeller approach was extended to yield a spherical hoop

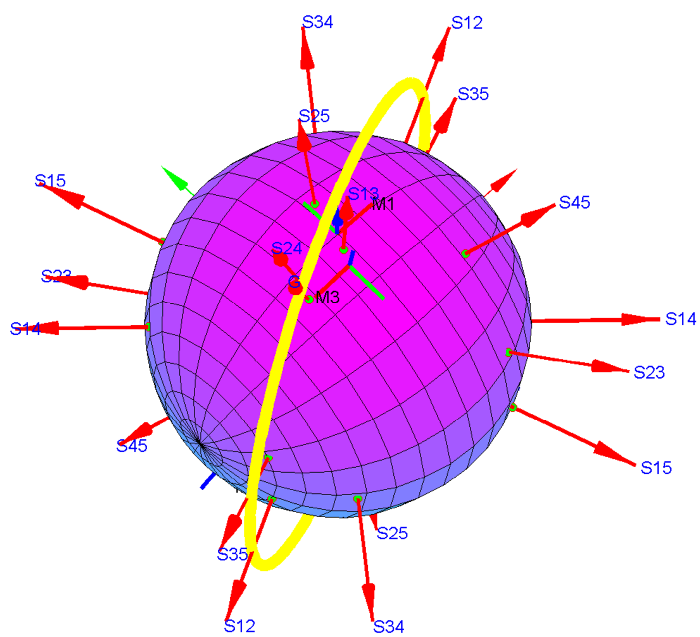

Figure 12. THE HOOP ON THE DESIGN SPHERE

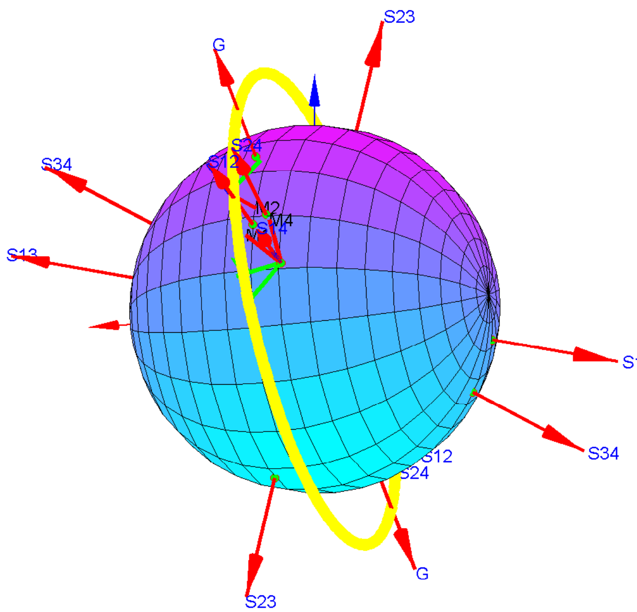

Figure 13. THE HOOP, ORIENTATIONS, ROTATION AXES, AND FIXED AXIS ON THE DESIGN SPHERE

methodology. The hoop is a great circle on the design sphere that intersects the fixed axis of the spherical RR dyad. The hoop method involves rotating the hoop about the fixed axis and noting the order in which the relative rotation axes are encountered. This method was shown to be useful for performing the order analysis of spherical RR dyads. Finally, the utility of the hoop method was demonstrated in three case studies. A MATLAB implementation of the hoop method for the orientation order analysis of spherical RR dyads is available upon request.

ACKNOWLEDGMENT

The authors acknowledge the contributions of Myszka, Murray‘, and Schmiedeler in creating the propeller method [9,10]; an elegant and effective tool for assessing position order of planar RR dyads.

The material presented here is based upon preliminary work reported in [8]. This material is based upon work supported by the National Science Foundation under grant #0422705. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation.

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