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ANALYSIS OF PLANAR RECONFIGURABLE MOTION GENERATORS

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ABSTRACT

This article presents the kinematic analysis of a new class of mechanical devices called planar Reconfigurable Motion Generators (RMGs) for multi-phase motion generation tasks having only a slight change in the desired motion. The concepts of configuration matrices and state vectors are introduced as a novel way to represent planar RMGs. The kinematic analysis of a planar RMG in it's initial configuration to determine the associated adjustable link lengths and angles is discussed. The adjustable link lengths are used to determine the link vector and state vector of a RMG. Future work will involve the application of configuration matrices and state vectors to aid in both systematic analysis and synthesis of RMGs.

INTRODUCTION

This paper presents the kinematic analysis of a new class of machines called Reconfigurable Motion Generators (RMGs) for multi-phase planar motion generation. RMGs are a new class of mechanical devices designed around a specific part family of products having only a slight variation and allow changes in their structure for motion planning tasks, Larochelle and Venkataramanujam [1].

Traditional hard automation linkage or cam driven mechanisms provide an extremely high speed capability at a relatively low cost. This is desirable for automation tasks that are not expected to change (i.e., hard). However in automation tasks where there is a requirement for flexibility in the motion generation tasks (i.e., soft), these machines are inadequate and call for the implementation of multi d.o.f. serial chain manipulators. Multi d.o.f. serial chain manipulators offer more flexibility and have the ability to adapt to a variety of divergent tasks. However, they are expensive to procure and maintain and have relatively longer cycle times. In most industrial situations the requirements for flexibility in operations are very limited. A typical serial chain robot may be only used for a few distinct pick-and-place operations throughout its operational life. This results in inadequate utilization of the robot. A discussion on the relative advantages and disadvantages of closed chain mechanisms and serial chain manipulators may be found in, Kota [2] and Larochelle and Venkataramanujam [1]. There is a need for a new class of mechanical devices that offer the speed of hard automation mechanisms along with some degree of the operational flexibility found in serial chain manipulators. Reconfigurable Motion Generators are a new class of mechanical devices conceived to address this need.

An arrangement of brakes at the passive R joints for link adjustment in a five-bar two d.o.f. mechanism in order to enable multi-phase motion generation has been proposed by Larochelle and Venkataramanujam [1]. It has been shown that for a five-bar mechanism with brakes instead of passive revolute joints we can obtain four distinct four-bar mechanisms or configurations. Formulations involving link geometry and joint configuration are developed to yield the adjustable link lengths and their associated joint parameters for a certain configuration. The analyses pre-

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sented here will enable the synthesis and design of planar RMGs for multi-phase motion generation tasks.

This paper proceeds as follows. First, the motivation and objectives of planar RMGs are presented. Next, the reconfiguration principle is discussed. This is followed by an introduction to planar RMGs and the reconfiguration principle for planar RMGs. The next section describes the configuration matrix and state vector that form the basis for the analysis of planar RMGs. RMGs are then analyzed in their initial configuration to compute the link vector.

BACKGROUND AND LITERATURE REVIEW

Four-bar mechanisms have been widely used for path, function, and motion generation tasks. However, single degree-of-freedom (d.o.f.) closed chain mechanisms like these can only perform a single motion generation task without disassembly. There is a need to redesign the mechanism even for a slight change in the desired motion generation task. Also single d.o.f. four-bar mechanisms are limited by the number of independent dimensional parameters that are available for synthesis. Single d.o.f. four-bar mechanisms also suffer from inherent drawbacks such as branch, order, and circuit defects that may arise in the synthesis process, Soh and McCarthy [3]. These limitations of single d.o.f. closed chain mechanisms can be overcome by enabling the adjustment of one or more structural parameters (link lengths or pivot locations) of the mechanism. Such mechanisms have been referred to as adjustable mechanisms or adjustable linkages, Tao [4]. Chuenchom and Kota [5] called these mechanisms "Programmable Mechanisms" or "Adjustable Robotic Mechanisms (ARMs)". Adjustable four-bar mechanisms can greatly enhance the ability to generate various kinds of output motions using the same set of links. They provide not only the flexibility required in many industrial applications, but also high operational speed, high load- bearing, and high precision capabilities. Thus, they form the true middle ground between conventional closed chain mechanisms and flexible serial chain manipulators, Chuenchom and Kota [5] and Kota [6].

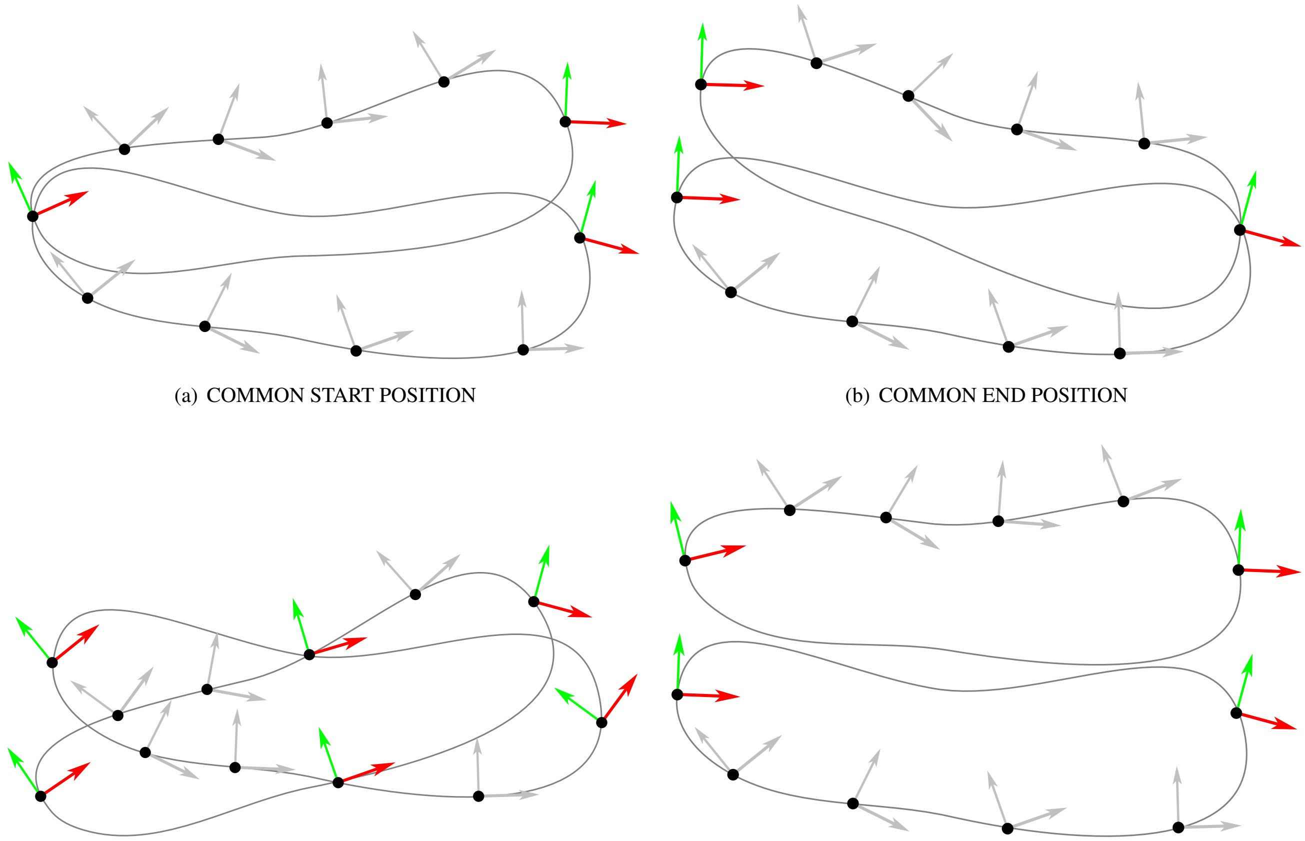
Several analytical or graphical methods for the analysis and synthesis of adjustable mechanisms have been proposed in the literature. McGovern and Sandor [7, 8] proposed the synthesis of function and path generating mechanisms by adjusting fixed pivot locations to attain two sets of precision points. Anees and Waldron [9] studied the synthesis of adjustable four-bar mechanisms by an adjustment of the position of the driven crank fixed pivots. Naik and Amarnath [10] presented a technique based on five-bar linkage theory to synthesize a four-bar adjustable function generator operating in two phases to produce two specified functions. Chang [11] proposed synthesis methods to design adjustable four-bar mechanisms for tracing variable circular arcs with prescribed velocities. Russel and Sodhi [12] presented a new technique for synthesizing planar four-bar mechanisms to achieve phases of both precise rigid body positions and rigid body positions with tolerances. Wang and Sodhi [13] studied the synthesis of adjustable moving pivot four-bar mechanisms. Peng and Sodhi [14] developed an optimal synthesis method for multi-phase continuous path generation of adjustable planar four-bar linkages. Lee et al. [15] demonstrated a method to synthesize adjustable spherical four-bar mechanisms for two-phase motion generation. Chanekar and Ghosal [16, 17] have dealt with optimization based method for synthesis for adjustable planar and spherical four-bar, crank-rocker mechanisms for multiple different and desired paths to be traced by a point on the coupler.

A graphical approach for the synthesis of adjustable linkages to generate variable coupler curves, either symmetrical or with cusps was proposed by Tao and Krishnamoorthy [18, 19]. Huston and Kramer [20] combined complex number methods and graphical techniques to synthesize adjustable planar four-bar mechanisms for symmetrical coupler curves. There has been research on methodologies for design of adjustable slider crank mechanisms for multi-phase motion, function and path generation by Zhou and Ting [21], Russel and Sodhi [22, 23]. Kim [24] discussed the idea of joint unactuation/actuation to adjust the manipulability and as a result improve the task adaptability of closed chain mechanisms. Kim and Choi [25] have discussed the idea of improved task adaptability of open/closed chain mechanisms through continuous joint mode conversion.

OBJECTIVE

The objective of this research effort is to investigate methodologies for the analysis, synthesis and manufacture of a new class of machines called Reconfigurable Motion Generators (RMGs). The objective of this article is to present the analysis of a planar RMG using the concept of RMG configuration matrices and state vectors. This work is part of an effort to develop low d.o.f. mechanisms capable of performing tasks that are currently performed by multiple d.o.f. serial chain manipulators. The concept of mechanism reconfiguration using active joints is used for improved task adaptability, in which the geometry of a mechanism is easily altered depending on the rigid body guidance task to be accomplished. RMGs are capable of generating distinct output motions using the same set of links. The adjustment of one or more parameters of the RMG facilitates multi-phase motion generation.

The utilization of active revolute joints for link length and pivot adjustment to achieve different one d.o.f. four-bar mechanisms from a single two d.o.f. five-bar mechanism is favorable as compared to the use of prismatic joints since revolute joints are relatively easy to manufacture and maintain. This methodology may be used for different types of motion generation tasks involving a common start, common end, common intermediate, and no common positions, shown in Figure 1(a), Figure 1(b), Figure 1(c), and Figure 1(d) respectively.



(a) COMMON START POSITION (b) COMMON END POSITION

(c) COMMON INTERMEDIATE POSITION (d) NO COMMON POSITIONS

FIGURE 1. **DIFFERENT TYPES OF MULTI-PHASE MOTION GENERATION TASKS**

This technique can be applied towards the synthesis of RMGs for use in many applications involving multi-phase function, path, and motion generation tasks. Finite position synthesis algorithms [26, 27] can be used to determine the dimensional parameters of the four-bar mechanism that can attain the desired motion generation tasks. The parameters of four-bar mechanisms that can attain the two sets of motion generation can be used to synthesize a RMG that can attain the two four-bar mechanisms as two states of the same configuration or two different configurations by locking/unlocking its joints as desired. This change may be achieved by a discrete locking/unlocking of the joints after the mechanism has performed one set of motion generation tasks in a desired configuration. In addition the RMG is stopped completely before any locking/unlocking operation is carried out.

The "Configuration" of a RMG is defined by the joint being locked. Consider the planar RMG shown in Figure 3. It may be observed that in this case joint C of the RMG is locked thus forming a one d.o.f four-bar mechanism OABD. Thus the RMG is said to be in configuration 3. The output link BD forms the adjustable link of the RMG in this configuration.

The "State" of a RMG is defined by the length of the adjustable link that subtends the locked joint. In Figure 3 the state of the RMG is defined by the magnitude (length) of the adjustable link BD that subtends the locked joint C.

RECONFIGURATION PRINCIPLE

In dimensional synthesis, the objective is to calculate the mechanism parameters required to achieve or approximate a set of prescribed rigid-body positions. This mechanism design objective is particularly significant when the rigid-body must achieve a prescribed set of end effector positions for a specific task e.g., a pick-and-place operation.

Consider the adjustable mechanism suggested for a pick-and-place operation by Chuenchom and Kota [28] shown in Figure 2. The figure shows two different types of parts being transported from conveyor 1. Depending on the type of part being transported from conveyor 1, the mechanism is required to place the part on conveyor 2 or conveyor 3. These two distinct pick-and-place operations comprise two different rigid body guidance or motion

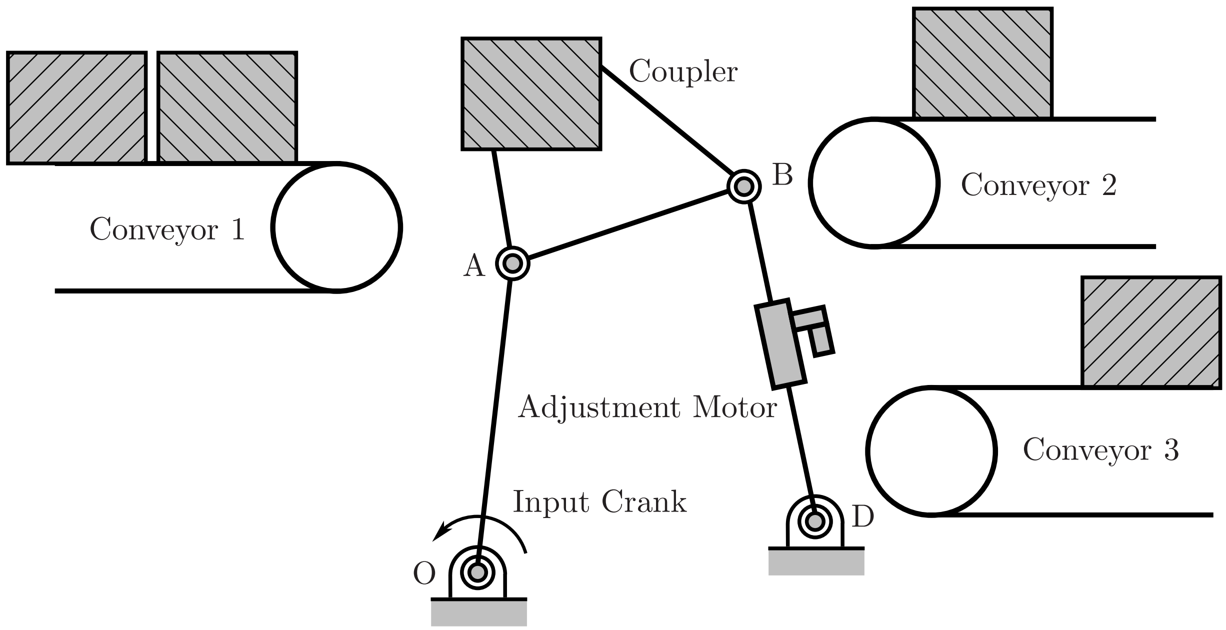


FIGURE 2. AN ADJUSTABLE PLANAR LINKAGE

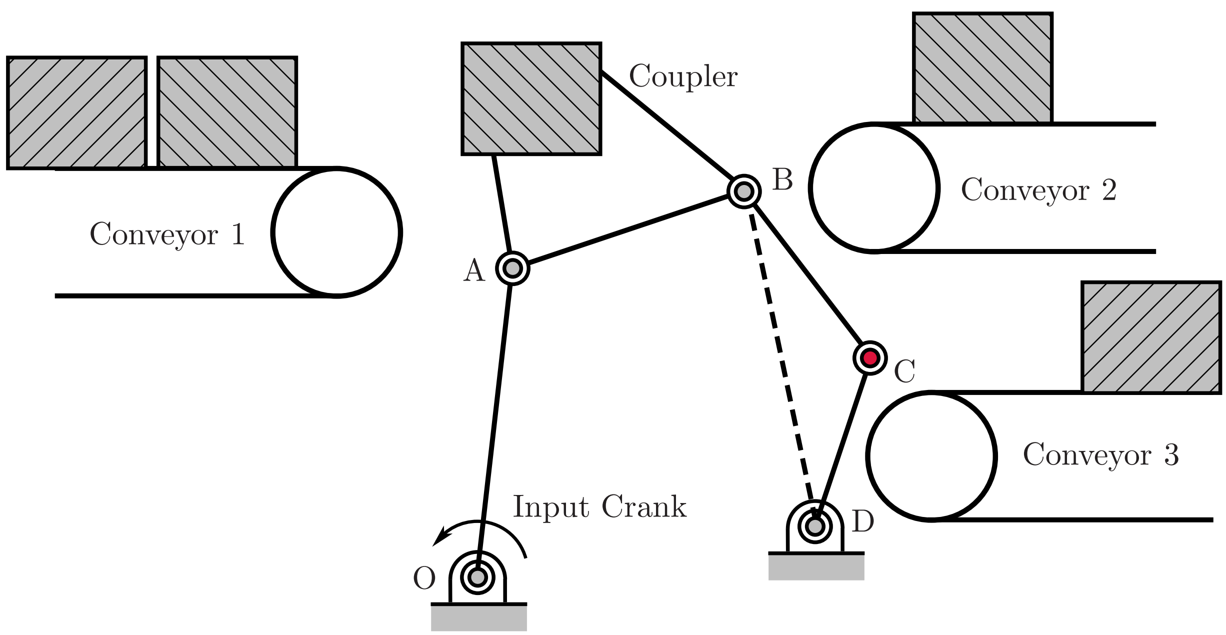


FIGURE 3. RECONFIGURABLE MOTION GENERATOR

generation tasks. The solution proposed by Chuenchom and Kota [28] for the task is an adjustable planar four-bar mechanism which can switch between the two tasks by a readjustment of the output link length by means of an adjustment motor. This gives a degree of adjustability to the mechanism and allows it to switch from one configuration to another. However, there are some drawbacks to this system due to the use of an additional actuator such as weight, cost, and complexity. The proposed alternative shown in Figure 3 is to replace the adjustable link (BD) by an RR dyad, see [1]. The resultant mechanism is a two

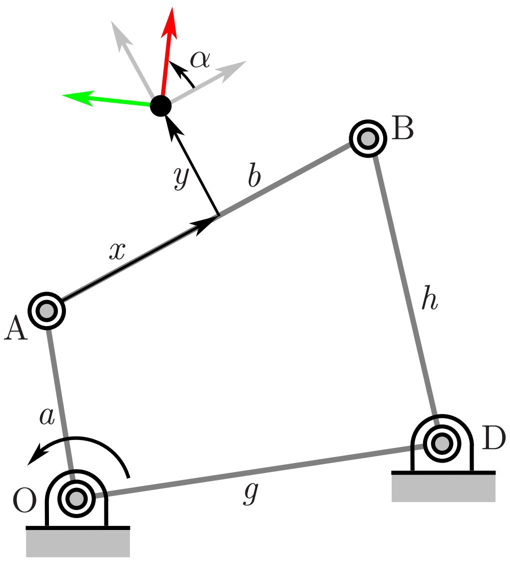


FIGURE 4. **FOUR-BAR MECHANISM PARAMETERS**

d.o.f. five-bar mechanism. If joint C on the five-bar mechanism is locked, e.g., by means of an electromagnetic clutch or brake, the five-bar mechanism loses one d.o.f. and is reduced to a four-bar mechanism with output link BD. The length of the resultant link BD can thus be adjusted by locking joint C at the desired position. Electromagnetic clutches and brakes can be used in joints where parameter adjustment is desired. This arrangement of links where brakes are incorporated at the joints has the inherent advantages of speed and ease of manufacture. Since the mechanism does not have a separate adjustment motor for readjustment of the link length it is expected to be significantly lighter due to lower mass of links and consequently facilitate higher speeds and lower inertia. These mechanisms are called Reconfigurable Motion Generators (RMGs), based on the definition of a reconfigurable mechanism as discussed by Kuo et al. [29].

BASE CONFIGURATION OF A RMG

Planar four-bar mechanisms may be used for multi-phase motion generation tasks by an adjustment of link lengths and pivot locations. The parameters that can be adjusted in a four-bar mechanism performing a motion generation task shown in Figure 4 are,

- Driving fixed pivot (O).
- Driven fixed pivot (D).
- Link lengths: input (a), output (h), and coupler (b)
- Coupler attachment: M (x, y, α).

In order to illustrate the concept of RMGs let us consider the five-bar mechanism OABCD shown in Figure 5. The mechanism

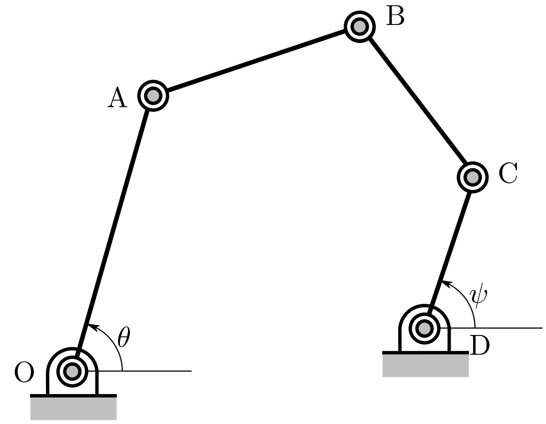


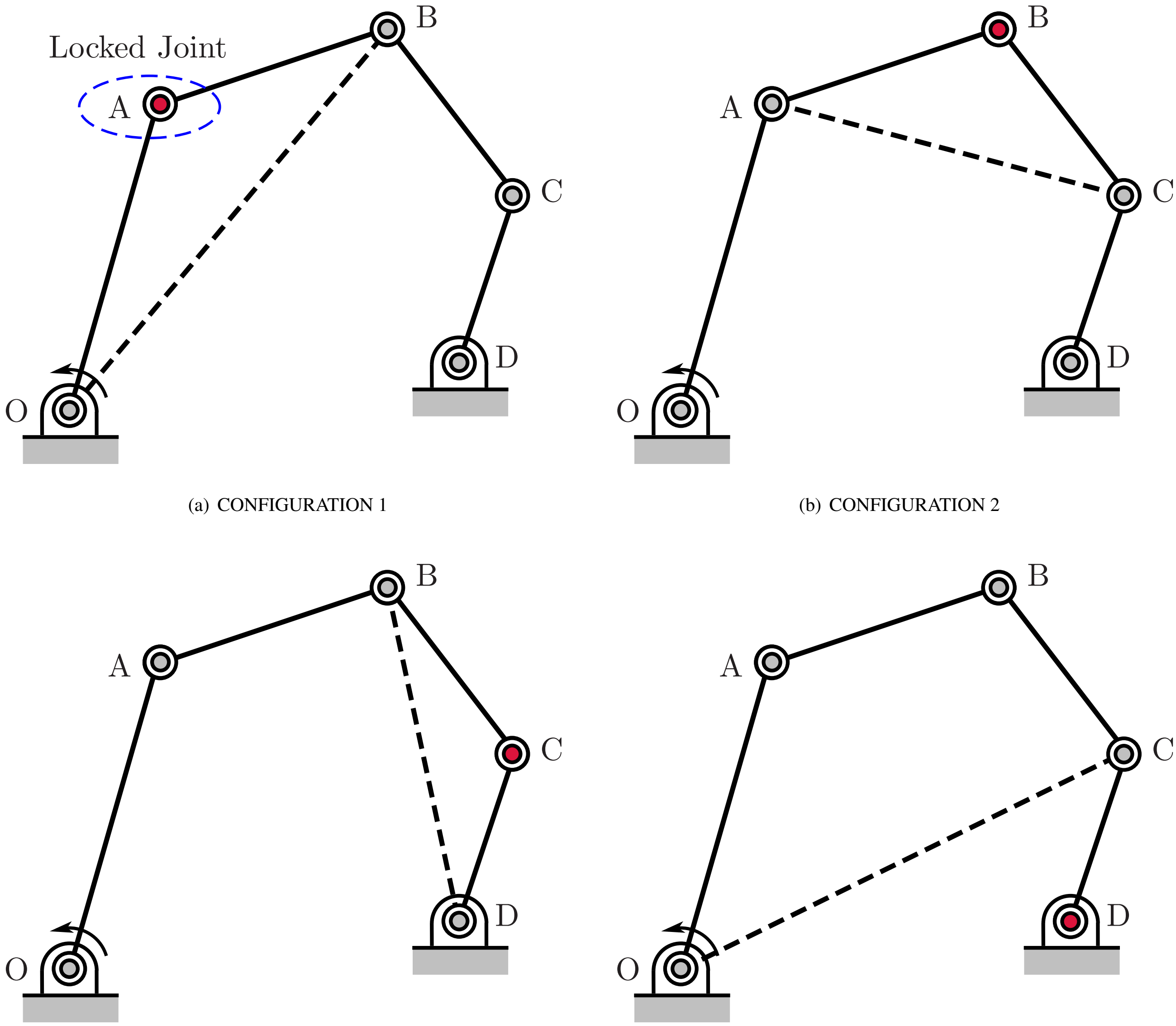
FIGURE 5. **RMG: BASE CONFIGURATION**

is a planar two d.o.f. five-bar mechanism which requires two independent inputs. This mechanism is called the base configuration of the RMG. The five-bar mechanism can be constrained in five different ways by locking or braking one of the revolute (R) joints to yield a planar one d.o.f. four-bar mechanism. The joints that can be locked are O, A, B, C, and D respectively giving us five different one d.o.f. four- bar mechanisms. Joint O forms the driving fixed pivot of the mechanism, thus it is never locked. Thus, we obtain four different configurations of the RMG based on the joint being locked.

CONFIGURATIONS OF A RMG

The planar RMG is a two d.o.f. five-bar mechanism that can be transformed into four different one d.o.f. four-bar mechanisms or configurations by locking one of the joints at A, B, C, or D by means of brakes as shown in Figure 6. The four different configurations of the RMG are summarized in Table 1, where LINK denotes the adjustable link. A single five-bar mechanism using one driving actuator and four relatively low cost electromagnetic brakes can be configured to perform multiple sets of motion generation tasks by a simple locking or braking of its passive R joints. The implementation of brakes at the joints allows the RMG to switch configurations without the need for manual adjustment. An RMG can perform distinct motion generation tasks in different configurations as well as different states in the same configuration.

Thus, if two distinct sets of motion generation tasks are to be achieved, four-bar mechanisms can be synthesized for each of the desired tasks. The two four-bar mechanisms thus obtained may be considered to be different configurations and states of the RMG that have to be achieved to attain the two different motion generation tasks. The two four-bar mechanisms thus become the starting points for the design of a RMG that can achieve both the desired motions by simply switching configurations and states.



(a) CONFIGURATION 1

(b) CONFIGURATION 2

(c) CONFIGURATION 3

(d) CONFIGURATION 4

FIGURE 6. DIFFERENT CONFIGURATIONS OF A RECONFIGURABLE MOTION GENERATOR

Thus, an RMG may be synthesized to achieve both the tasks by switching on/off the brakes at the appropriate joints.

Configuration 1

Consider the planar RMG in configuration 1, shown in Figure 6(a). In this configuration joint A is locked and link OB forms the adjustable link of the RMG. Thus, the original two d.o.f. five-bar mechanism is transformed into a one d.o.f. four-bar mechanism OBCD. The length of the input link OB may be changed by controlling the position at which joint A is locked. This parameter adjustment can be realized by means of a brake at joint A.

Configuration 2

Consider the planar RMG in configuration 2, shown in Figure 6(b). In this configuration joint B is locked and link AC forms the adjustable link of the RMG. Thus, the original two d.o.f. five-bar mechanism is transformed into a one d.o.f. four-bar mechanism OACD. The length of the adjustable link AC may be changed by controlling the position at which joint B is locked.

TABLE 1. LINK PARAMETERS FOR RMG CONFIGURATIONS

	Link Parameter			
Config #.	Input	Output	Coupler	Fixed
1	<u>OB</u>	CD	BC	OD
2	OA	CD	<u>AC</u>	OD
3	OA	<u>BD</u>	AB	OD
4	OA	BC	AB	<u>OC</u>

TABLE 2. CONFIGURATIONS OF A PLANAR RMG

Configuration #.	O	A	B	C	D
Configuration 0	0	0	0	0	0
Configuration 1	0	1	0	0	0
Configuration 2	0	0	1	0	0
Configuration 3	0	0	0	1	0
Configuration 4	0	0	0	0	1

Configuration 3

Consider the planar RMG in configuration 3, shown in Figure 6(c). In this configuration joint C is locked and link BD forms the adjustable link of the RMG. Thus, the original two d.o.f. five-bar mechanism is transformed into a one d.o.f. four-bar mechanism OABD. The length of the output link BD may be changed by controlling the position at which joint C is locked.

Configuration 4

Similarly, consider the planar RMG in configuration 4, shown in Figure 6(d). In this configuration joint D is locked and link OC becomes forms the adjustable link of the RMG. Thus, the original two d.o.f. five-bar mechanism is transformed into a one d.o.f. four-bar mechanism OABC. The length of the fixed link OC may be changed by controlling the position at which joint D is locked. The link parameters for different configurations of the RMG are summarized in Table 1.

CONFIGURATION MATRIX AND STATE VECTOR

The number of configurations of a planar RMG are determined by,

$$^{n}C_r = \frac{r!}{r!(n-r)!} \tag{1}$$

where,
n is the number of active R joints, and
r is the number of joints being locked.
The number of configurations of a planar RMG based on a two d.o.f. five-bar mechanism is given by,

$$\begin{aligned} ^{4}C_1 &= \frac{4!}{1!(4-1)!} \\ &= 4 \text{ configurations} \end{aligned} \tag{2}$$

Similarly, the concept can be extended to a two d.o.f. six-bar mechanism driven by one actuator, and two active brakes. The number of configurations in this RMG is computed to be,

$$\begin{aligned} ^{5}C_2 &= \frac{5!}{2!(5-2)!} \\ &= 10 \text{ configurations} \end{aligned} \tag{3}$$

Configuration Matrix

The configurations of a planar RMG are shown in Table 2, where the columns represent the joints and rows represent configurations. The state of an active joint (i.e., passive R joint replaced by an electromagnetic brake) in a RMG is represented by,

$$\mathrm{S} = \begin{cases} 1: & \text{On (Locked)} \\ 0: & \text{Off (Free)} \end{cases} \tag{4}$$

From Table 2 it can be seen that in Configuration 0 (Base Configuration), all the joints are unlocked, this is a two d.o.f. five-bar mechanism which forms the basis for a planar RMG. In configuration 1, 2, 3, and 4, brakes are switched on at joints A, B, C, and D respectively to form distinct one d.o.f. four-bar mechanisms or configurations. The state of each joint in a planar RMG forms the basis for the Configuration Matrix (**CM**) defined by the equation,

$$\mathbf{CM} = \begin{bmatrix} \mathrm{S(A)} & 0 & 0 & 0 \\ 0 & \mathrm{S(B)} & 0 & 0 \\ 0 & 0 & \mathrm{S(C)} & 0 \\ 0 & 0 & 0 & \mathrm{S(D)} \end{bmatrix} \tag{5}$$

In the RMG configuration matrix the row and column numbers correspond to the configuration numbers and joints in a planar RMG respectively. Let us consider the case in which joint A is locked, i.e., RMG is in configuration 1. Here,

$$\begin{aligned} \mathrm{S(A)} &= 1 \\ \mathrm{S(B)},\ \mathrm{S(C)},\ \mathrm{S(D)} &= 0 \end{aligned} \tag{6}$$

Thus the Configuration Matrix is determined to be,

$$\mathbf{CM} = \begin{bmatrix} 1\,0\,0\,0 \\ 0\,0\,0\,0 \\ 0\,0\,0\,0 \\ 0\,0\,0\,0 \end{bmatrix} \tag{7}$$

The Link Vector ($\mathbf{LV}$) is represented by the lengths of the adjustable links that subtend the locked joint. Thus the link vector is represented by,

$$\mathbf{LV} = \begin{bmatrix} k_A\ k_B\ k_C\ k_D \end{bmatrix}^{\mathsf{T}} \tag{8}$$

State Vector

The State Vector ($\mathbf{SV}$) which defines the current configuration and state of the RMG is given by,

$$\mathbf{SV} = \mathbf{CM} \times \mathbf{LV} \tag{9}$$

For example, if k_A represents adjustable link length of the RMG in configuration 1, the state vector of the RMG is given by,

$$\begin{aligned} \mathbf{SV} &= \mathbf{CM} \times \mathbf{LV} \\ &= \begin{bmatrix} 1\,0\,0\,0 \\ 0\,0\,0\,0 \\ 0\,0\,0\,0 \\ 0\,0\,0\,0 \end{bmatrix} \times \begin{bmatrix} k_A \\ k_B \\ k_C \\ k_D \end{bmatrix} \\ &= \begin{bmatrix} k_A \\ 0 \\ 0 \\ 0 \end{bmatrix} \end{aligned} \tag{10}$$

The state vector thus completely defines a RMG's state and configuration by virtue of two parameters, i.e., the joint being locked and length of the adjustable link that subtends the locked joint. Consider the following case in which a RMG needs to switch from an adjustable link length $k_A i$ in configuration 1 to an adjustable link length $k_C f$ in configuration 3. Thus the link vectors for the initial and final states may be represented by $[k_A\ k_B\ k_C\ k_D]_i^{\mathsf{T}}$ and $[k_A\ k_B\ k_C\ k_D]_f^{\mathsf{T}}$. The state vector of the RMG in configuration 1 and adjustable link length $k_A i$ may thus be represented by,

$$\begin{bmatrix} 1\,0\,0\,0 \\ 0\,0\,0\,0 \\ 0\,0\,0\,0 \\ 0\,0\,0\,0 \end{bmatrix} \times \begin{bmatrix} k_A \\ k_B \\ k_C \\ k_D \end{bmatrix}_i = \begin{bmatrix} k_A \\ 0 \\ 0 \\ 0 \end{bmatrix}_i \tag{11}$$

Similarly, the state vector of the RMG in configuration 3 and adjustable link length $k_C f$ may thus be represented by,

$$\begin{bmatrix} 0\,0\,0\,0 \\ 0\,0\,0\,0 \\ 0\,0\,1\,0 \\ 0\,0\,0\,0 \end{bmatrix} \times \begin{bmatrix} k_A \\ k_B \\ k_C \\ k_D \end{bmatrix}_f = \begin{bmatrix} 0 \\ 0 \\ k_C \\ 0 \end{bmatrix}_f \tag{12}$$

ANALYSIS OF A RMG IN INITIAL CONFIGURATION

In order to determine the Link Vector ($\mathbf{LV}$) of the RMG in its initial (starting) configuration we proceed as follows. In its original configuration the RMG is a one d.o.f freedom mechanism since the output fixed pivot is locked. In this configuration the input (θ) and output (ψ) angles are known, see Figure 7. The link vector is formed by determining the lengths of the adjustable links that subtend all the active joints on the RMG as follows.

The analysis is started from the output fixed pivot (D) of the RMG. Consider $\triangle$ODC, here

$$\sigma_D = \pi - \psi \tag{13}$$
$$\lambda_D = \arcsin[(g/k_D)\sin\sigma_D]$$
$$\varepsilon_D = \arcsin[(h/k_D) sin \sigma_D]$$
$$k_D = \sqrt{g^2 + h^2 - 2gh\cos(\sigma_D)} \tag{14}$$

Consider $\triangle$OAC, here

$$\delta_O + \lambda_A = \theta - \varepsilon_D$$
$$k_B = \sqrt{a^2 + k_D^2 - 2ak_D\cos(\delta_O + \lambda_A)} \tag{15}$$
$$\delta_C = \arcsin[(a/k_B)\sin(\delta_O + \lambda_A)]$$
$$\delta_A = \arcsin[(k_D/k_B)\sin(\delta_O + \lambda_A)]$$

Consider $\triangle$ABC, here

$$\sigma_B = \arccos[(b^2 + c^2 - k_B^2)/2bc] \tag{16}$$
$$\lambda_B = \arcsin[(c/k_B)\sin(\sigma_B)]$$
$$\varepsilon_B = \arcsin[(b/k_B)\sin(\sigma_B)]$$

Consider $\triangle$OAB, here

$$\sigma_A = \lambda_B + \delta_A \tag{17}$$
$$k_A = \sqrt{a^2 + b^2 - 2ab\cos(\sigma_A)} \tag{18}$$
$$\varepsilon_A = \arcsin[(a/k_A)\sin(\sigma_A]$$
$$\lambda_A = \arcsin[(b/k_A)\sin(\sigma_A)]$$

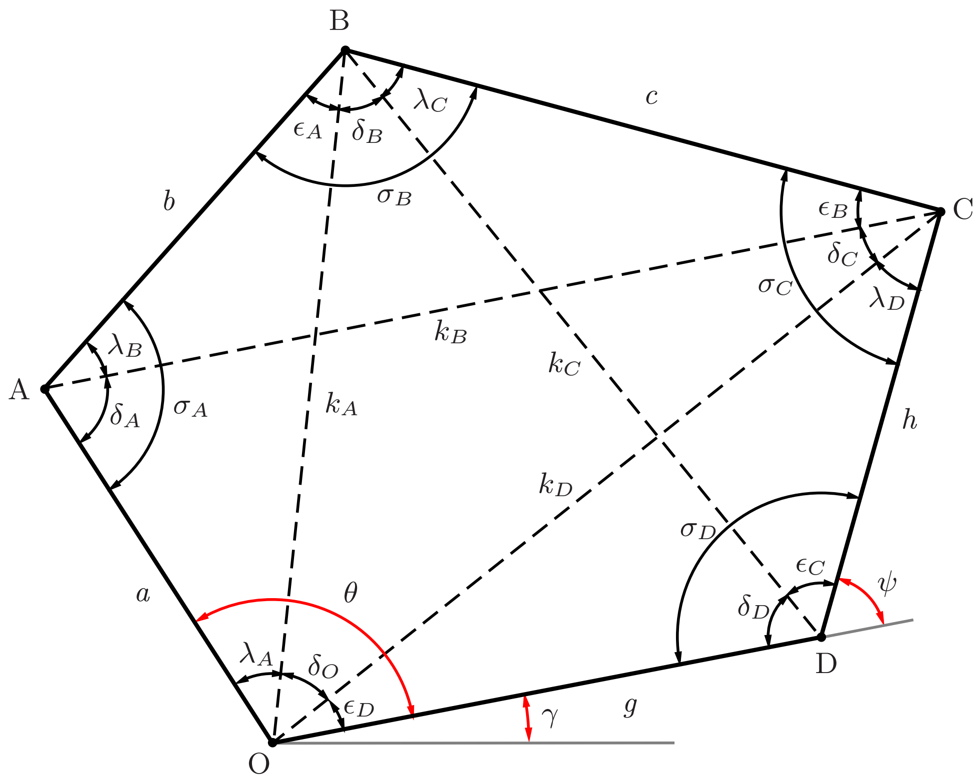


FIGURE 7. RMG IN INITIAL CONFIGURATION

Consider △BCD, here

$$\sigma_C = \varepsilon_B + \delta_C + \lambda_D \tag{19}$$

$$k_C = \sqrt{c^2 + h^2 - 2ch\cos(\sigma_C)} \tag{20}$$

The link parameters are determined from Equations.14, 15, 18, and 20. Thus, in the initial configuration of a planar RMG the known input angles θ and ψ together with the link parameters can be used to determine the state of the RMG including the adjustable link lengths at the subtended joints. The adjustable link lengths form the components of the link vector that may be used for further analysis and synthesis of a planar RMG.

CONCLUSION

This paper introduces a novel methodology for the kinematic analysis of planar Reconfigurable Motion Generators. In this article, the position analysis of a planar RMG has been presented. A Configuration Matrix and a State Vector to describe the geometry and kinematic structure of a RMG are defined. The adjustable link lengths and subtended angles at the joint are computed analytically to determine the Link Vector. The Link Vector is used in conjunction with the Configuration Matrix to determine the State Vector. The State Vector represents the configuration as well as the state of a RMG. This vector may be further used to represent changes in configurations and states of RMGs. Future work will involve application of these concepts to aid in a systematic analysis and synthesis of RMGs for multi-phase motion generation tasks.

ACKNOWLEDGEMENT

This work is based on preliminary research reported by Larochelle and Venkataramanujam [1, 30] that introduces the concept and basic principles of RMGs.

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