Synthesis of Spatial CC Dyads and 4C Mechanisms for Pick & Place Tasks with Guiding Locations

P. Larochelle

Abstract A novel dimensional synthesis technique for solving the mixed exact and approximate motion synthesis problem for spatial CC kinematic chains is presented. The methodology uses an analytic representation of the spatial CC dyad's rigid body constraint equation in combination with an algebraic geometry formulation of the perpendicular screw bisector to yield designs that exactly reach the prescribed pick & place locations while approximating an arbitrary number of guiding locations. The result is a dimensional synthesis technique for mixed exact and approximate motion generation that utilizes only algebraic geometry and does not require the use of any iterative optimization algorithms or a metric on spatial displacements. An example that demonstrates the synthesis technique is included.

Key words: Spatial Mechanisms, 4C Mechanisms, CC Dyads

1 Introduction

As a product is assembled in an automated factory a common task that needs to be performed is the movement of parts or subassemblies from one location to another; this is commonly referred to as a *pick & place* task. For the assembly of a complex product the number of pick & place tasks that need to be performed could run into the thousands. Parts are picked out of bins and placed into subassemblies, subassemblies are picked up and placed into the final product, etc. One solution is to use devices with a high number of degrees of freedom such as industrial robots. Robots can perform these tasks but at penalties in costs, cycle time, and maintenance. A second solution is to use a cascading series of simple one degree of freedom de-

Robotics & Spatial Systems Lab
Department of Mechanical and Aerospace Engineering
Florida Institute of Technology, e-mail: pierrel@fit.edu

vices; e.g. a series of servo motors. Creating such a manipulation pipelines takes a longer design time and is often more art than science.

Spatial robotic mechanisms offer another alternative. The synthesis algorithm presented here is part of ongoing efforts directed at realizing the capability to design two degree of freedom robotic spatial mechanisms capable of performing spatial pick & place tasks. These low degree of freedom devices are capable of producing the necessary spatial motion for accomplishing pick & place tasks. Hence, spatial robotic mechanisms provide an alternative for solving spatial assembly tasks that might otherwise require a robot or multiple single degree of freedom devices.

A well known result from screw theory [9, 1] is that moving an object from one spatial location to another doesn't require six degrees of freedom. In fact, such motions can be accomplished with a single degree of freedom twist about a unique screw axis. However this solution is often impractical due to the location of the screw axis within the workspace and the collisions and interferences between objects that may result. Spatial robotic mechanisms are low degree of freedom machines that are a compromise between the 6 or more degree of freedom industrial robot and the series of single degree of freedom motion generators. Here, we focus on utilizing the spatial CC dyad as the motion generator for a class of spatial robotic mechanisms to achieve two desired locations exactly (i.e. pick & place) while approximating a set of guiding locations that take the workpiece from the pick location to the place location.

In a related work [12] presents the derivation of the constraint manifold for spherical RR dyads using the image space representation of displacements. This work was an extension of the ideas presented in [13]. In [11] the spatial generalization of the planar Burmester curves was presented from a geometric viewpoint. The focus of this work was the synthesis of CC and related dyads for exact motion generation through three and four locations. The synthesis of CC dyads for exact motion through 5 locations was presented in [10, 9]. In [6, 3, 5] the extension of Burmester threory, using Roth's line congruence approach [13], for the exact synthesis of 4C mechanisms for 4 locations is presented. The approximate motion synthesis of spatial 4C mechanisms for rigid body guidance was presented in [7]. Circuit and branch defects of the spatial 4C mechanism were investigated in [4] and the detection of self-collisions of the links was discussed in [2]. The methodology used here for performing the dimensional synthesis for mixed exact and approximate rigid body guidance is based upon the works of [14] and builds upon the spherical version presented in [8].

This paper proceeds as follows. First, the geometry and kinematics of the spatial CC dyad are reviewed. Next, the synthesis algorithm for solving the mixed exact and approximate motion generation problem for spatial CC dyads is presented. Finally, an example spatial robotic mechanism design is presented; the synthesis of a spatial 4C mechanism to accomplish a pick & place tasks exactly while approximating three guiding locations.

2 Synthesis Algorithm

A spatial 4C closed chain may be viewed as the combination of two CC dyads where each dyad consists of one link and two C joints; one fixed and the other moving, see Fig. 1. The approach taken here is to synthesize two dyads separately and then join their floating links to yield a kinematic closed chain. Let the fixed axis be specified by the dual vector $\hat{\mathbf{u}}$ measured in the fixed reference frame F and let the moving axis be specified by $\hat{\mathbf{v}}$ measured in the moving frame M. Moreover, let $\hat{\mathbf{l}}$ define the moving axis $\hat{\mathbf{v}}$ in the fixed frame F so that, $\hat{\mathbf{l}} = [\hat{A}]\hat{\mathbf{v}}$ where $[\hat{A}]$ is the dual orthogonal matrix that defines M with respect to F [9]. Because the link is rigid, the dual angle between the two axes of the dyad remains constant. This geometric constraint may be expressed analytically as,

$$\hat{\mathbf{u}} \cdot \hat{\mathbf{l}} = \hat{\mathbf{u}} \cdot [\hat{A}]\hat{\mathbf{v}} = \cos\hat{\alpha}. \qquad (1)$$

This constraint equation is the foundation of the synthesis algorithm presented below. In order to solve the mixed exact and approximate synthesis problem we first solve the exact synthesis problem for 3 prescribed locations.

2.1 Exact Synthesis for Three Locations

Here we select a moving axis $\hat{\mathbf{v}}$ of a CC dyad and solve for the corresponding fixed axis $\hat{\mathbf{u}}$ such that the dyad guides the moving body exactly through 3 prescribed locations [6]. To solve this synthesis problem we first work with the real or direction part of the CC constraint equations and then subsequently address the moment part. We write the real part of Eq. 1 for each of the desired locations, $[\hat{A}]_i, i = 1,2,3$. Next, we subtract the first equation from the remaining two to arrive at a linear system of equations,

$$[P]\mathbf{u} = \mathbf{k} \qquad (2)$$

where,

$$[P] = \begin{bmatrix} (\mathbf{l}_2 - \mathbf{l}_1)^T \\ (\mathbf{l}_3 - \mathbf{l}_1)^T \\ 0\ 0 \quad 1 \end{bmatrix},$$

$\mathbf{l}_i$ is the direction of the moving axis in the i^{th} location, $\mathbf{k} = [0\ 0\ 1]^T$, and $\mathbf{u}$ is the desired direction of the fixed axis. Note that we must solve Eq. 2 for each moving axis direction to find its corresponding fixed axis direction. Moreover, note that since we are using 3-vectors to define the axes when in fact they are directions that only require 2 independent coordinates, the last row of $[P]$ is chosen to yield the vector $\mathbf{u}$ that is the intersection of the fixed axis with the $z = 1$ plane. In the event that $[P]$ is rank deficient (i.e. when the fixed axis does not intersect the $z = 1$ plane) simply

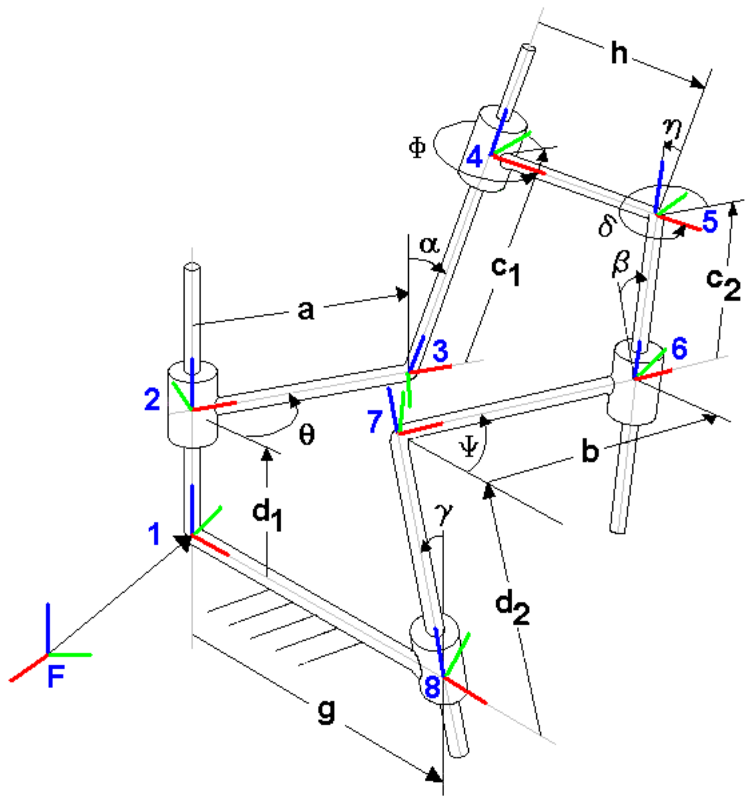

Fig. 1 Spatial 4C Mechanism: Geometry & Nomenclature.

change the last row to any vector that does not lie in this plane (e.g. $[1\ 0\ 0]^T$). Next, we proceed to solve for the moment of the fixed axis.

We write the dual part of Eq. 1 for each of the desired locations, $[\hat{A}]_i, i = 1,2,3$ and then subtract the first equation from the remaining two to arrive at a linear system of equations,

$$[H]\mathbf{u}^0 = \mathbf{t} \tag{3}$$

where,

$$[H] = \begin{bmatrix} (\mathbf{l}_2 - \mathbf{l}_1)^T \\ (\mathbf{l}_3 - \mathbf{l}_1)^T \\ \mathbf{u}^T \end{bmatrix}, \quad \mathbf{t} = \begin{bmatrix} -(\mathbf{l}_2^0 - \mathbf{l}_1^0)^T \mathbf{u} \\ -(\mathbf{l}_3^0 - \mathbf{l}_1^0)^T \mathbf{u} \\ 0 \end{bmatrix},$$

and $\mathbf{u}^0$ is the desired moment of the fixed axis. Solve Eqs. 2 & 3 for each desired moving axis of a CC dyad to find the unique corresponding fixed axis that guides the moving body exactly through the 3 prescribed locations.

2.2 Mixed Synthesis Algorithm

We now consider the synthesis of CC dyads that guide a moving body exactly through 2 pick & place locations and approximately through n guiding locations. First a desired moving axis $\hat{\mathbf{v}}$ is selected. Next, we seek a corresponding fixed axis $\hat{\mathbf{u}} = (\mathbf{u}, \mathbf{u}^0)$ for the dyad. We proceed by identifying the spherical image of the CC dyad. Duffy showed that associated with each spatial CC dyad there is a spherical image consisting of a spherical RR dyad whose link lengths are the angular twists of the CC dyad. Moreover, he proved that the spatial CC dyad and its associated spherical RR image have the exactly the same angular relationships and motions [1, 9]. Therefore the synthesis of the spatial CC dyad can be decomposed into two subproblems; (1) the angular synthesis or the synthesis of the link twist angles of the CC dyad and (2) the moment synthesis or the synthesis of the link length of the CC dyad. We address the former first.

The angular synthesis of the spatial CC dyad can be solved by performing the synthesis of its spherical RR image. The direction of the fixed axis is found by solving n 3 orientation problems to yield a set of fixed axis directions $\mathbf{u}_i, i = 1, 2, \ldots, n$. The 3 orientation problems are derived from the 2 pick & place locations along with 1 of the guiding locations. Hence, there are n unique 3 orientation problems (Eq. 2) that are solved to obtain n fixed axis directions $\mathbf{u}_i, i = 1, 2, \ldots, n$. It was shown in [8] that the direction of the fixed axis that will guide the moving body as desired is the normalized sum of these directions of $\mathbf{u}_i, i = 1, 2, \ldots, n$,

$$\mathbf{u} = \frac{\sum \mathbf{u}_i}{\|\sum \mathbf{u}_i\|}. \tag{4}$$

We now focus on the moment synthesis problem; finding the desired moment $\mathbf{u}^0$ of the fixed axis $\hat{\mathbf{u}}$.

The moment synthesis of the spatial CC dyad can be solved by utilizing the geometric interpretation of Eq. 1; that $\hat{\mathbf{u}}$ must lie on the screw perpendicular bisector associated with the pick & place locations of the desired moving axis $\hat{\mathbf{v}}$. For the CC dyad to reach exactly the pick & place locations Eq. 1 must hold true in both locations. Write Eq. 1 for the pick & place locations and take the difference to yield,

$$\hat{\mathbf{u}} \cdot (\hat{\mathbf{l}}_{\mathbf{place}} - \hat{\mathbf{l}}_{\mathbf{pick}}) = 0. \tag{5}$$

Eq. 5 is the equation of the screw perpendicular bisector of $\hat{\mathbf{l}}_{\mathbf{pick}}$ and $\hat{\mathbf{l}}_{\mathbf{place}}$ [9]. The set of screws $\hat{\mathbf{u}}$ that satisfy Eq. 5 is a two parameter set whose axes intersect and are orthogonal to $\hat{\mathbf{B}}$ as shown in Fig. 2. Note that $\hat{\mathbf{N}}$ is the common normal to $\hat{\mathbf{l}}_{\mathbf{pick}}$ and $\hat{\mathbf{l}}_{\mathbf{place}}$, $\hat{\mathbf{V}}$ is the midpoint screw, and $\hat{\mathbf{B}} = \hat{\mathbf{N}} \times \hat{\mathbf{V}}$. Recall that the direction of $\hat{\mathbf{u}}$ has been previously found from Eq. 4. Therefore finding a point on the fixed axis $\hat{\mathbf{u}}$ is sufficient for determining the unknown moment $\mathbf{u}^0$. From the properties of the screw perpendicular bisector it is known that $\hat{\mathbf{u}}$ must intersect and be orthogonal to $\hat{\mathbf{B}}$; we now determine this point of intersection and use it to determine the unknown moment $\hat{\mathbf{u}}^{\mathbf{0}}$.

For the prescribed moving axis $\hat{\mathbf{v}}$ solve n 3 location problems to yield a set of fixed axes $\hat{\mathbf{u}}_i, i = 1, 2, \ldots, n$. The 3 location problems are derived from the 2 pick & place locations along with 1 of the guiding locations. Hence, there are n unique 3 location problems (Eqs. 2 & 3) that are solved to obtain n fixed axes $\hat{\mathbf{u}}_i, i = 1, 2, \ldots, n$. Because each of these CC dyads guide the body exactly through the pick & place locations their fixed axes also intersect $\hat{\mathbf{B}}$. Note that if each of these n CC dyads exactly reach all of the guiding locations then their n fixed axes $\hat{\mathbf{u}}_i, i = 1, 2, \ldots, n$ intersect $\hat{\mathbf{B}}$ in a unique point. In general the CC dyads will not be capable of exactly reaching the n guiding locations and the intersections of their fixed axes with $\hat{\mathbf{B}}$ will not be a unique point. Next, determine these n intersection points $\mathbf{p}_i, i = 1, 2, \ldots, n$. The desired point $\mathbf{p}$ on the fixed axis $\hat{\mathbf{u}}$ is the average of these intersection points,

$$\mathbf{p} = \frac{\sum \mathbf{p}_i}{n}. \tag{6}$$

Finally the unknown moment may be determined from $\mathbf{u^0} = \mathbf{p} \times \mathbf{u}$. The CC dyad with prescribed moving axis $\hat{\mathbf{v}}$ and fixed axis $\hat{\mathbf{u}}$, as determined with the above algorithm, guides the moving body exactly through the pick & place locations and near the n guiding locations.

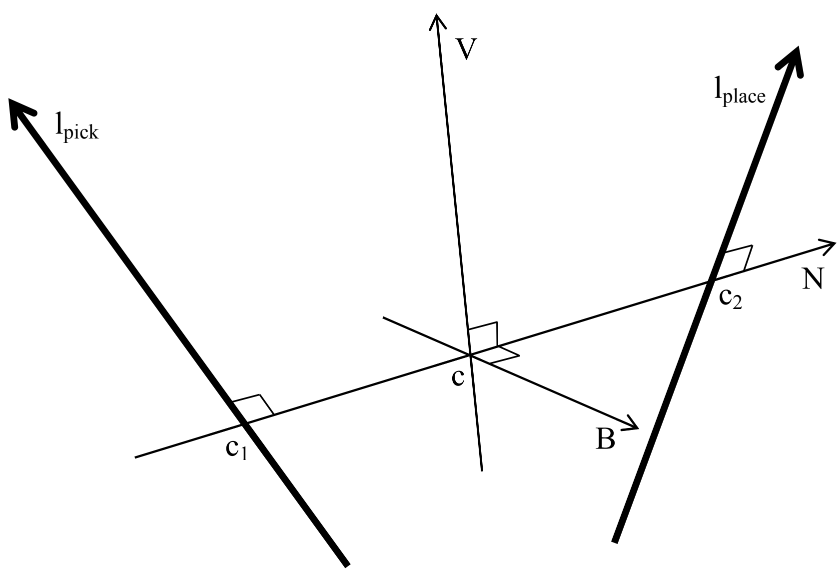

Fig. 2 Spatial CC Dyad & The Screw Perpendicular Bisector.

3 Example

We employ the preceding methodology and design a 4C spatial mechanism to guide a moving body exactly through two pick & place locations and near 3 guiding locations as defined in Tb. 1 where $[A] = [Rot_z(lng)][Rot_y(-lat)][Rot_x(rol)]$ and all angles are expressed in degrees. Two CC dyads are synthesized independently and then their floating links are joined to yield a 4C closed-chain mechanism.

For dyad #1 a moving axis was prescribed: $\hat{\mathbf{v}}_1 = [0.2673\ 0.5345\ -0.8018\ 0.0000\ 0.8018\ 0.5345]^T$. The mixed synthesis algorithm presented above yielded $\mathbf{u}_1 = [-0.1292\ 0.4342\ 0.8915]^T$, $\mathbf{p}_1 = [0.7935\ \ -0.0109\ \ -1.1956]^T$, and fixed axis $\hat{\mathbf{u}}_1 = [-0.1292\ 0.4342\ 0.8915\ 0.5094\ -0.5529\ 0.3431]^T$. The resulting CC dyad's link lengths are: $a = -2.34$ and $\alpha = 121.15$ (deg). For dyad #2 a different moving axis was chosen: $\hat{\mathbf{v}}_2 = [0.5774\ -0.5774\ 0.5774\ -0.5774\ -0.5774\ 0.0000]^T$. The mixed synthesis algorithm yielded $\mathbf{u}_2 = [-0.6675\ 0.5265\ 0.5265]^T$, $\mathbf{p}_2 = [2.5000\ \ -1.0968\ \ 0.0968]^T$, and fixed axis $\hat{\mathbf{u}}_2 = [-0.6675\ 0.5265\ 0.5265\ -0.6284\ -1.3809\ 0.5842]^T$. The resulting CC dyad's link lengths are: $b = 4.70$ and $\beta = 112.67$ (deg). When the two dyads are combined to form a spatial 4C mechanism the fixed link length is $g = 0.72$ and $\gamma = 38.34$(deg) and the length of the coupler link is $h = 0.78$ and $\eta = 128.11$(deg). This 4C mechanism has a non-Grashof $0-\pi$ double-rocker spherical four-bar image [4, 1].

To verify the motion of the moving body the CC dyad constraint equations were evaluated in each of the 5 locations; the left-hand side of Eq. 1, i.e $\hat{\mathbf{u}} \cdot [\hat{A}]\hat{\mathbf{v}}$, is reported in the right columns of Tb. 1. Note that the inner product between the fixed and moving lines of each CC dyad is identical in the pick & place locations thereby verifying that the moving body does in fact reach the pick & place locations exactly.

Table 1 Five Prescribed Locations & Synthesis Results.

Longitude	Latitude	Roll	X	Y	Z	Motion Type	Dyad #1 Constraint	Dyad #2 Constraint
0.00	0.00	0.00	3	5	-1	exact	$-0.5173+2.0058\varepsilon$	$-0.3854-4.3362\varepsilon$
0.00	25.00	10.00	4	4	-2	approximate	$-0.8074+1.9092\varepsilon$	$-0.1617-5.3040\varepsilon$
20.00	45.00	20.00	2	3	-3	approximate	$-0.8760+2.0811\varepsilon$	$-0.3371-4.6082\varepsilon$
65.00	65.00	10.00	5	2	-4	approximate	$-0.5801+2.2399\varepsilon$	$-0.5922-2.7565\varepsilon$
90.00	90.00	0.00	1	1	-5	exact	$-0.5173+2.0058\varepsilon$	$-0.3854-4.3362\varepsilon$

4 Conclusions

A novel dimensional synthesis technique for solving the mixed exact and approximate motion problem for spatial CC open and 4C closed kinematic chains has been presented. The methodology uses an analytic representation of the spatial CC dyad's rigid body constraint equation in combination with classical geometric motion synthesis techniques to yield designs that exactly reach two prescribed pick & place locations while approximating *n* guiding locations. Such tasks are common in automated assembly and production systems. An example was presented to demonstrate the synthesis procedure.

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