Synthesis of Planar Mechanisms for Pick and Place Tasks With Guiding Positions

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A novel dimensional synthesis technique for solving the mixed exact and approximate motion synthesis problem for planar RR kinematic chains is presented. The methodology uses an analytic representation of the planar RR dyad's rigid body constraint equation in combination with an algebraic geometry formulation of the exact synthesis for three prescribed positions to yield designs that exactly reach the prescribed pick and place positions while approximating an arbitrary number of guiding positions. The result is a dimensional synthesis technique for mixed exact and approximate motion generation for planar RR dyads. A solution dyad may be directly implemented as a 2R open chain or two solution dyads may be combined to form a planar 4R closed chain, also known as a planar four-bar mechanism. The synthesis algorithm utilizes only algebraic geometry and does not require the use of a numerical optimization algorithm or a metric on elements of SE(2); the group of planar displacements. Two implementations of the synthesis algorithm are presented; computational and graphical construction. Moreover, the kinematic inversion of the algorithm is also included. Two examples that demonstrate the synthesis technique are included. [DOI: 10.1115/1.4028638]

Introduction

An algorithm for synthesizing planar 4R closed-chain mechanisms, also referred to as planar four-bar mechanisms, is presented. The planar 4R closed chain consists of four links, one link being fixed, connected in series by revolute joints (R) to form a 1 deg of freedom closed kinematic chain. A planar 4R closed chain may be viewed as the combination of two planar RR dyads; where each dyad consist of three links, one link being fixed, connect by two R joints, see Fig. 1. The synthesis objective here is to generate a planar 4R mechanism that guides a moving body exactly through two positions (referred to as pick and place) and near n positions (referred to as guiding positions). The synthesis algorithm presented utilizes an analytic representation of the planar RR dyad's rigid body constraint equation along with classical geometric constructions for exact motion synthesis. In addition, a measure of the quality of the approximation to the n guiding positions is presented. The result is an effective synthesis procedure that is geometrically intuitive, fast to implement, and does not involve the use of any nonlinear optimization algorithms. Note that while the focus here is on using two RR dyads to form a planar four-bar mechanism that solves the mixed and exact motion generation problem, the synthesis algorithm may also be used to generate solution RR dyads that may be combined to yield other kinematic topologies such as geared five-bar mechanisms, six-bar mechanisms, and eight-bar mechanisms.

In related works Mirth presented an algorithm for the design of planar four-bar mechanisms based upon synthesis algorithms that yield solutions that guide a body through four positions. The synthesis of planar RR dyads for four exact positions is a well known problem whose solution involves the center point and circle point curves of classical Burmester theory [1,2]. Mirth exploited the known properties of these curves to yield algorithms for two exact [3,4] and three exact [5] positions and n approximate positions. Mirth solved multiple four position problems to yield multiple center or circle point curves. Graphically, he identified regions of acceptable solutions from these curves. Holte et al. extended that work with a mathematical method for identifying the regions of acceptable solutions [6]. In Ref. [7] the method was extended to include approximate velocity constraints. It is important to note that Mirth's algorithm includes specific tolerance bounds for the approximate task positions while the algorithm presented here does not include such bounds.

Sutherland [8] presents a mixed exact-approximate synthesis technique based upon the geometric constraint of the RR dyads;

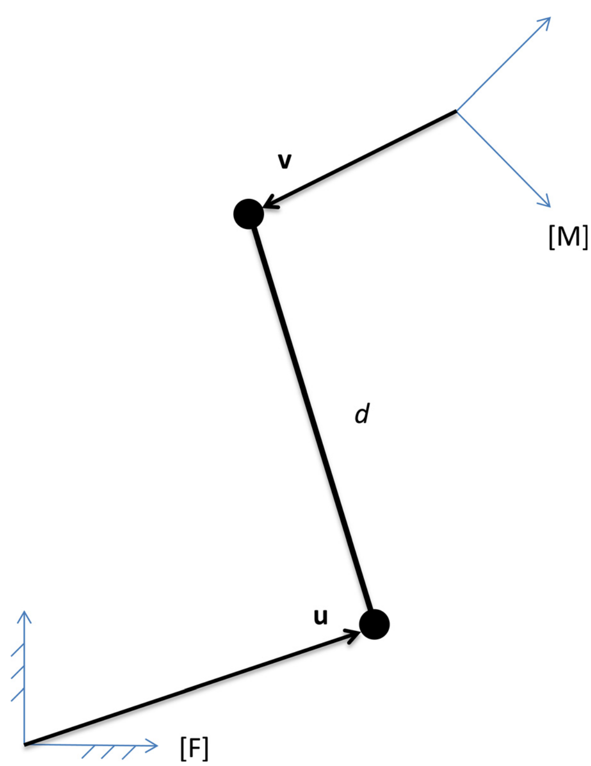

Fig. 1 Planar RR dyad geometry and nomenclature

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that the moving pivot travel on a circle centered at the fixed pivot. He utilized the implicit algebraic representation of the circle along with an iterative solution algorithm to minimize the square of the dyad's link length; i.e., a least-squares minimization problem. Sutherland addresses the one, two, three, and four exact position problems. Kramer and Sandor [9] utilize nonlinear programming techniques to synthesis dyads that approximate the desired positions. Their selective precision methodology enables the designer to define accuracy neighborhoods around each precision position. Smaili and Diab [10] present the mixed exact-approximate synthesis of planar mechanisms for point path tasks. They use logarithmic terms in the objective function of their nonlinear optimization problem so that some points may be approximated while others are nearly reached exactly. Luu and Hayes [11] present integrated type and dimensional synthesis of planar mechanisms for rigid body guidance that uses numerical methods to determine both the mechanism type and its approximate dimensions. The methodology used here for performing the dimensional synthesis for mixed exact and approximate rigid body guidance is based upon the foundational works of [12] and [2]. The general spatial case of the planar synthesis algorithm presented here can be found in Ref. [13], the spherical version in Ref. [14], and the planar version was first introduced in Ref. [15].

This paper proceeds as follows. First, the geometry and kinematics of the planar RR dyad and the planar 4R closed chains are reviewed. Next, the synthesis algorithm for solving the mixed exact and approximate motion synthesis problem for planar RR dyads is presented. Finally, two examples are included which illustrate the application of the methodology to the synthesis of planar four-bar mechanisms to achieve two prescribed positions exactly while approximating three (ex. #1) and nine (ex. #2) guiding positions.

Synthesis Algorithm

A planar 4R closed chain is viewed as the combination of two planar RR dyads where each dyad consists of two R joints: one fixed and the other moving, see Fig. 1. The approach taken here is to separately synthesize two dyads and then join their floating links to yield a closed kinematic chain. Let the fixed axis be specified by the vector $\mathbf{u}$ measured in the fixed reference frame F and let the moving axis be specified by $\mathbf{v}$ measured in the moving frame M. Moreover, let $\mathbf{l}$ define the moving axis $\mathbf{v}$ in the fixed frame F, so that $\mathbf{l} = [A]\ \mathbf{v} + \mathbf{d}$ where $[A]$ is the element of SO(2) that defines the orientation of M with respect to F and $\mathbf{d}$ is the translation vector from the origin of F to the origin of M. Because the link is rigid the linear distance d between the two joint axes of the dyad remains constant. This geometric constraint may be expressed analytically as

$$(\mathbf{u} - \mathbf{l}) \cdot (\mathbf{u} - \mathbf{l}) = (\mathbf{u} - [A]\mathbf{v} - \mathbf{d}) \cdot (\mathbf{u} - [A]\mathbf{v} - \mathbf{d}) = d^2 \quad (1)$$

This constraint equation is the foundation of the synthesis algorithm presented below. In order to solve the mixed exact and approximate synthesis problem we first solve the exact synthesis problem for three prescribed positions.

Exact Synthesis for Three Positions. First, we select the moving axis $\mathbf{v}$. Second, we write Eq. (1) for each of the desired positions, $([A]_i, \mathbf{d}_i)$ $i = 1, 2, 3$. Finally, we subtract the first equation from the remaining two to arrive at a linear system of equations

$$[P]\mathbf{u} = \mathbf{b} \quad (2)$$

where

$$[P] = \begin{bmatrix} 2(\mathbf{l}_2^{\mathrm{T}} - \mathbf{l}_1^{\mathrm{T}}) \\ 2(\mathbf{l}_3^{\mathrm{T}} - \mathbf{l}_1^{\mathrm{T}}) \end{bmatrix}$$

$$\mathbf{b} = \begin{pmatrix} \mathbf{l}_2^{\mathrm{T}}\mathbf{l}_2 - \mathbf{l}_1^{\mathrm{T}}\mathbf{l}_1 \\ \mathbf{l}_3^{\mathrm{T}}\mathbf{l}_3 - \mathbf{l}_1^{\mathrm{T}}\mathbf{l}_1 \end{pmatrix}$$

and $\mathbf{u}$ is the desired fixed axis. We must solve Eq. (2) for each prescribed moving axis to find its corresponding fixed axis. The result is a planar RR dyad that guides a rigid body precisely through three prescribed positions.

Mixed Synthesis Algorithm. In the problem considered here, we have two positions to reach exactly (i.e., pick and place) and n positions that serve to guide the body as it moves from the pick position to the place position.

First, it is beneficial to discuss the geometry underlying this approach. Consider the synthesis of a planar RR dyad for two exact positions for a desired moving axis $\mathbf{v}$. From Eq. (1), it is evident that a fixed axis $\mathbf{u}$ compatible with the two exact positions must be simultaneously equidistant to the moving axis in both positions; $\mathbf{l}_{\text{pick}}$ and $\mathbf{l}_{\text{place}}$. The set of all such points is a line; the perpendicular bisector of the line segment $\overline{\mathbf{l}_{\text{pick}}\mathbf{l}_{\text{place}}}$ [2].

Now consider the exact three position problem for a desired moving axis $\mathbf{v}$. The desired fixed axis $\mathbf{u}$ lies at the intersection of three perpendicular bisectors; the first associated with positions #1 and #2, the second with positions #2 and #3, and the third with #1 and #3. Generally, these three lines intersect in one point; hence, there is one unique fixed axis $\mathbf{u}$ associated with three planar positions and a prescribed moving axis $\mathbf{v}$.

Next, we progress to the mixed synthesis algorithm. Consider a pick and place task with $n = 3$ guiding positions. Recognizing that the desired fixed pivot $\mathbf{u}$ must lie on the perpendicular bisector associated with $\mathbf{v}$ and the pick and place positions we proceed by solving the three exact position synthesis problem n times; once for each guiding position, i.e., positions pick-1-place, pick-2-place, and pick-3-place. Each three position problem yields a fixed pivot $\mathbf{u}$ which we refer to by their associated guiding position numbers; $\mathbf{u}_1$, $\mathbf{u}_2$, and $\mathbf{u}_3$. Note that each fixed pivot lies on the perpendicular bisector associated with the pick and place positions and therefore each solution dyad will guide the moving body exactly through these two positions. The fixed pivot $\mathbf{u}$ that will guide the moving body exactly through the pick and place positions and moves nearest the n guiding positions must lie on this perpendicular bisector. The fixed pivot $\mathbf{u}_1$ yields a dyad that reaches positions pick-1-place exactly while $\mathbf{u}_2$ reaches positions pick-2-place exactly and $\mathbf{u}_3$ reaches positions pick-3-place. Note that if the dyad defined by $\mathbf{u}$ and $\mathbf{v}$ reached all five positions exactly then we have $\mathbf{u} = \mathbf{u}_1 = \mathbf{u}_2 = \mathbf{u}_3$. We utilize the fixed axis that lies on the perpendicular bisector associated with the two exact positions and that is nearest the perpendicular bisectors associated with the guiding positions. This fixed axis yields an RR dyad that has minimal structural error in each of the n guiding positions; where structural error is defined as the variation in the Euclidean distance between the fixed axis and the moving axis in each of the n guiding positions. Hence, the solution dyad will guide the moving body exactly through the two prescribed positions and near the guiding positions for the selected moving axis. The desired fixed pivot $\mathbf{u}$ that guides the moving body exactly through the pick and place positions and moves nearest the n guiding positions is the point on the perpendicular bisector that is nearest $\mathbf{u}_1$, $\mathbf{u}_2$, and $\mathbf{u}_3$. Next, we discuss how we implement this synthesis algorithm computationally.

The mixed synthesis algorithm may be implemented computationally as follows. First, a desired moving axis $\mathbf{v}$ is selected. Next, we seek a corresponding fixed axis for the dyad. The fixed axis is found by solving n three position problems (Eq. (2)) to yield a set of fixed axes $\mathbf{u}_i$, $i = 1, 2,\ldots, n$. Each of the three position problems is generated by combining the two exact positions (pick & place) along with 1 of the n guiding positions. Hence, we obtain n unique three position problems that are solved using Eq. (2). For the desired moving axis $\mathbf{v}$ the corresponding fixed

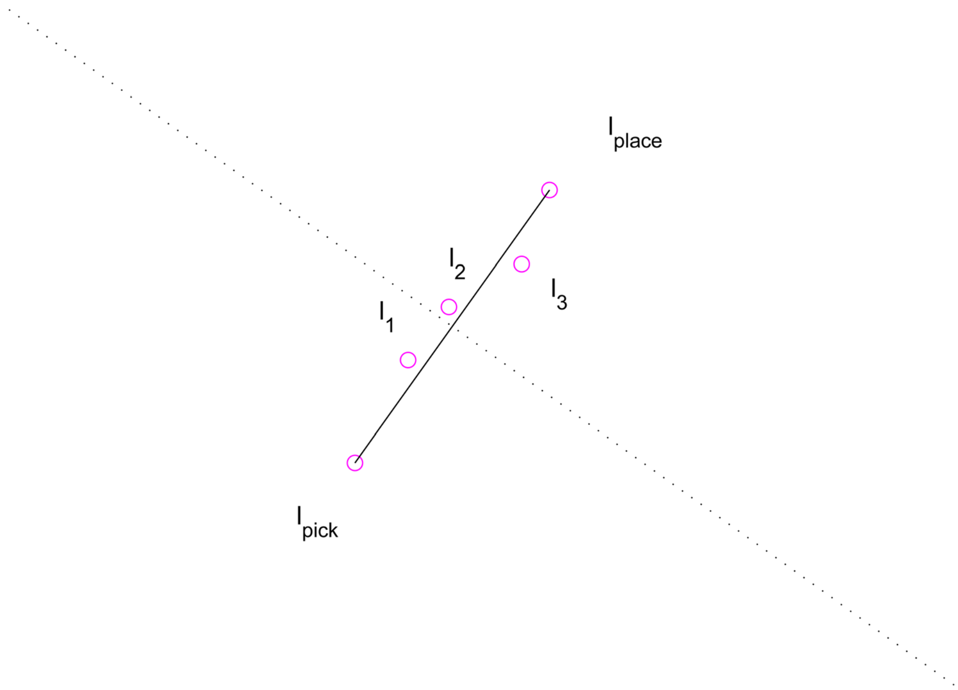

Fig. 2 Dyad #1

axis $\mathbf{u}$ that guides the moving body such it exactly reaches the pick and place positions and moves nearest the guiding positions is the arithmetic mean of $\mathbf{u}_i$

$$\mathbf{u} = \frac{\sum \mathbf{u}_i}{n} \tag{3}$$

Figures 2 and 3 illustrate the implementation of the mixed synthesis algorithm for a pick and place task with three guiding positions. Figure 2 illustrates the pick and place positions of the chosen moving pivot along with the associated perpendicular bisector. Also shown are the positions of the chosen moving pivot in each of the three guiding positions. Figure 3 follows Fig. 2 and adds the three fixed pivots that are determined by solving the associated three position problems; $\mathbf{u}_1$, $\mathbf{u}_2$, and $\mathbf{u}_3$ as well as the final solution fixed pivot $\mathbf{u}$ as determined by Eq. (3).

Step-by-Step: Geometric Construction. Here we present a step-by-step geometrical construction implementation of the synthesis algorithm.

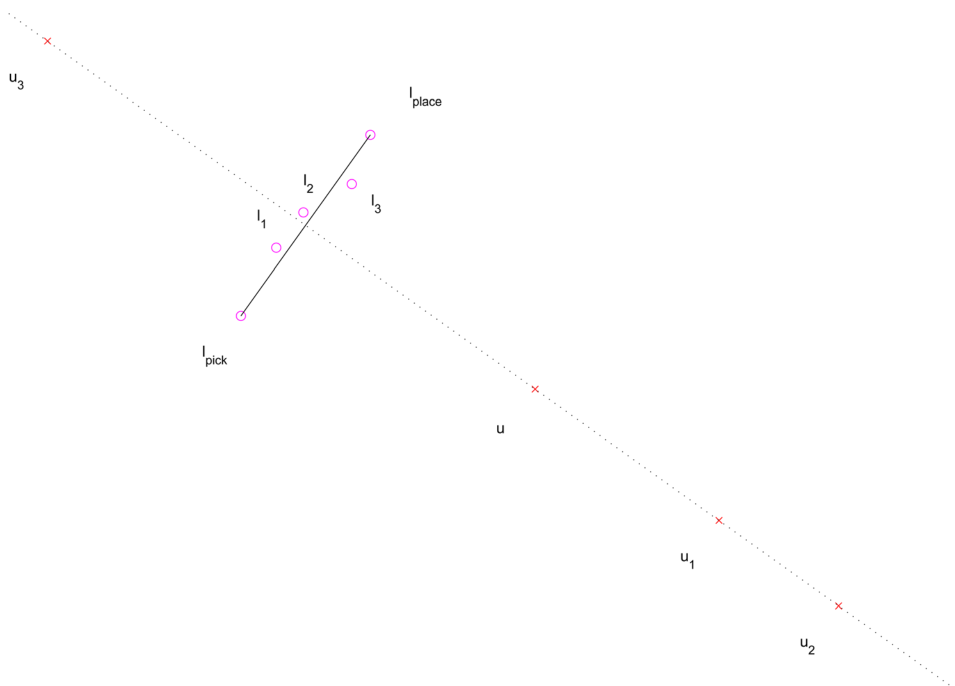

Fig. 3 Dyad #1

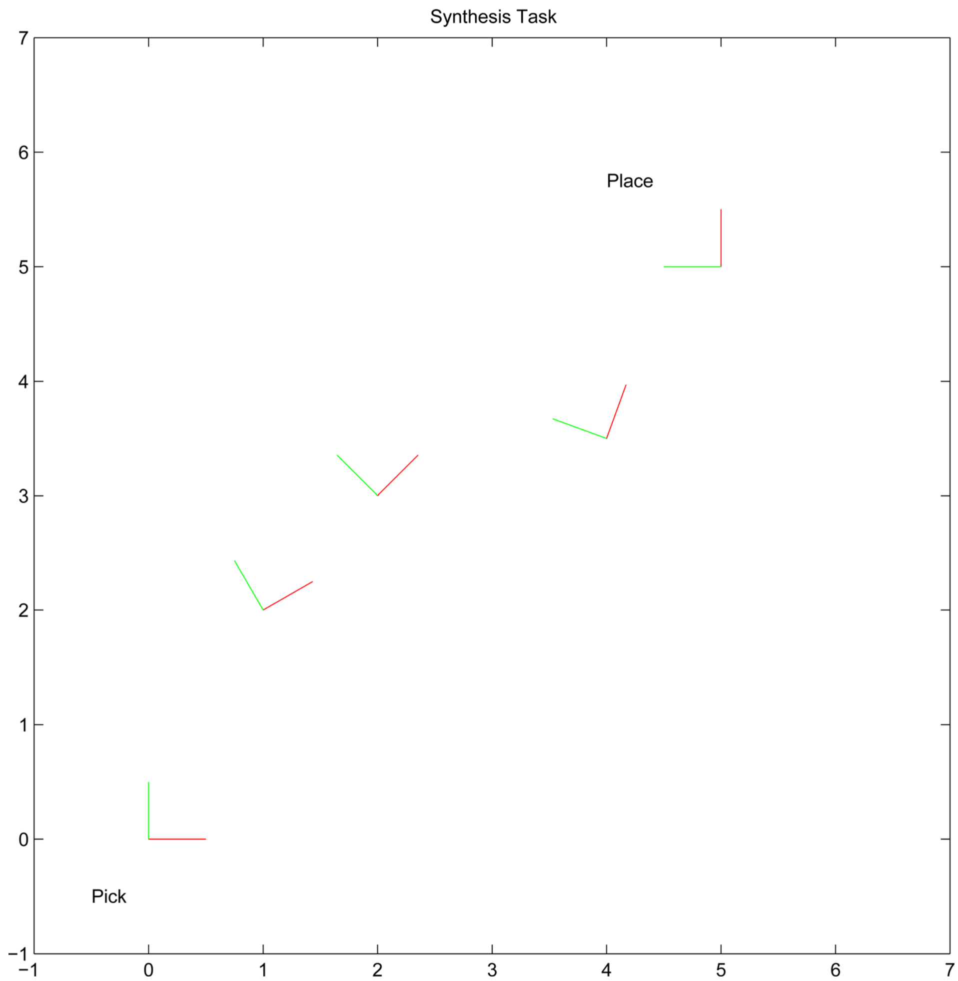

Fig. 4 Five prescribed positions

Problem Statement: Synthesize a planar RR dyad to exactly reach pick & place positions while moving the body nearest n guiding positions.
Given: Two exact positions (pick & place), n guiding positions, and choice of moving pivot $\mathbf{v}$.
Find: The fixed pivot $\mathbf{u}$ to yield a planar RR dyad that exactly reaches the pick & place positions while moving the body nearest the n guiding positions.

(1) Use trilateration to determine the position of the prescribed moving pivot $\mathbf{v}$ in each desired position: e.g., $\mathbf{l}_{\text{pick}}$, $\mathbf{l}_{\text{place}}$, $\mathbf{l}_1$, $\mathbf{l}_2,\ldots, \mathbf{l}_n$.
(2) For each guiding position i solve the three position synthesis problem [2] as follows. The solution to each three position problem (i.e., the fixed pivot $\mathbf{u}_i$) is the intersection of the perpendicular bisectors of $\overline{\mathbf{l}_{\text{pick}}\mathbf{l}_i}$ and $\overline{\mathbf{l}_{\text{pick}}\mathbf{l}_{\text{place}}}$.
(3) Using vector addition (head to tail) sum the vectors $\mathbf{u}_i$, $i = 1, 2,\ldots, n$. The desired fixed pivot $\mathbf{u}$ is the intersection of this vector sum and the perpendicular bisector of $\overline{\mathbf{l}_{\text{pick}}\mathbf{l}_{\text{place}}}$.

Step-by-Step: Computational Procedure. Here we present a Step-by-Step computational implementation of the synthesis algorithm.

Problem Statement: Synthesize a planar RR dyad to exactly reach pick & place positions while moving the body nearest n guiding positions.
Given: Two exact positions (pick & place), n guiding positions, and choice of moving pivot $\mathbf{v}$.
Find: The fixed pivot $\mathbf{u}$ to yield a planar RR dyad that exactly reaches the pick & place positions while moving the body nearest the n guiding positions.

(1) Determine the position of the prescribed moving pivot $\mathbf{v}$ in each desired position: e.g., $\mathbf{l}_{\text{pick}}$, $\mathbf{l}_{\text{place}}$, $\mathbf{l}_1$, $\mathbf{l}_2,\ldots, \mathbf{l}_n$.
(2) Solve Eq. (2) n times to yield a set of fixed axes $\mathbf{u}_i$, $i = 1, 2,\ldots, n$.
(3) Solve Eq. (3) to yield the desired fixed axis $\mathbf{u}$.

Approximation Quality Measure

When synthesizing a mechanism it is useful to know the degree to which the n guiding positions are being reached by each planar RR dyad. We propose the following approximation quality measure $\odot$, which is based upon the structural error in each of the guiding positions:

Table 1 Five prescribed positions

#	x	y	θ (deg)	Motion type
1	0.0	0.0	0.0	Exact
2	1.0	2.0	30.0	Approximate
3	2.0	3.0	45.0	Approximate
4	4.0	3.5	70.0	Approximate
5	5.0	5.0	90.0	Exact

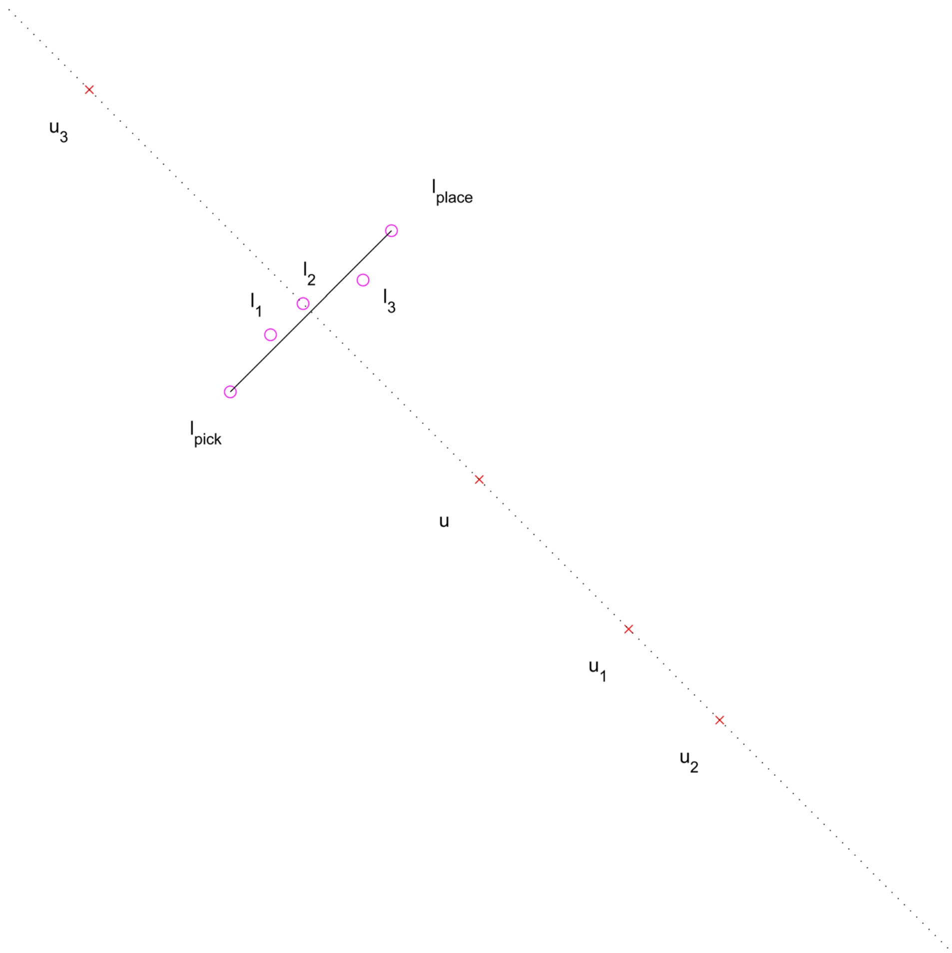

Fig. 5 Dyad #2

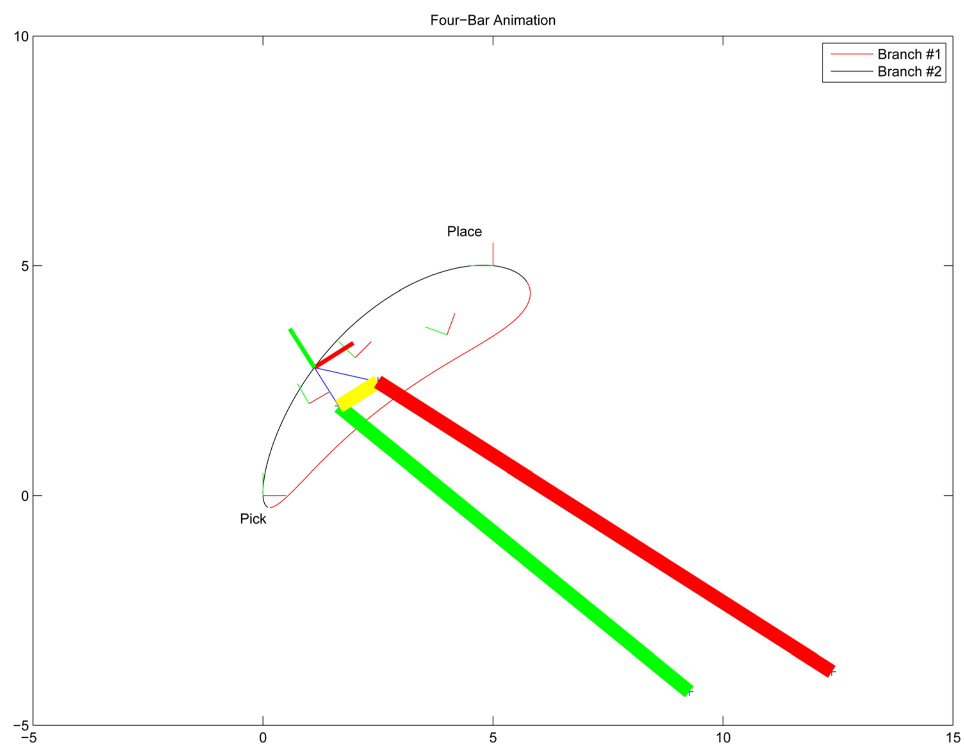

Fig. 6 The solution mechanism

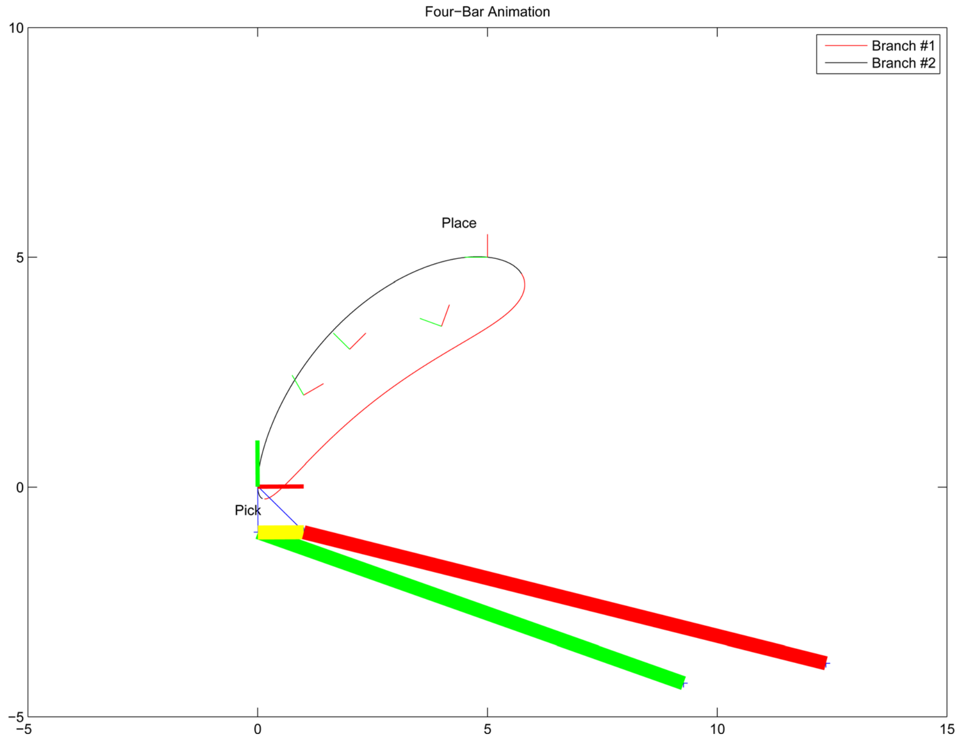

Fig. 7 The mechanism shown in the pick position

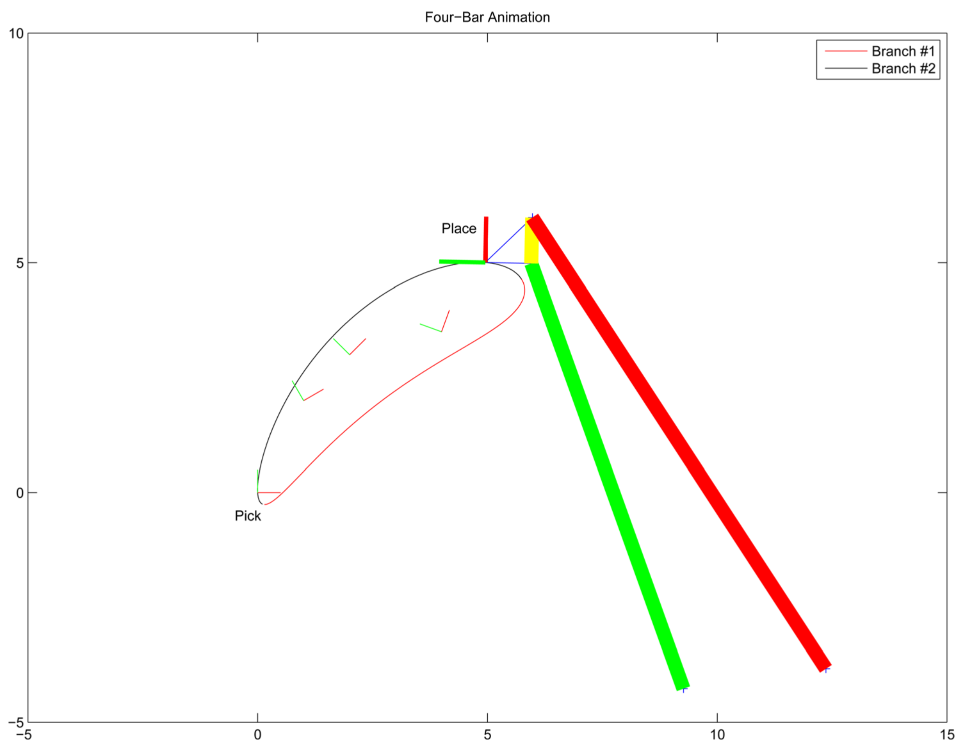

Fig. 8 The mechanism shown in the place position

$$\odot = 100 - \sum_{i=1}^{n} \|\mathbf{u}_i - \mathbf{u}\| \qquad (4)$$

If the dyad moves the workpiece such that it exactly reaches each of the guiding positions, then each vector difference in Eq. (4) yields **0** and therefore the 2-norm of the sum yields 0. Hence, if the dyad exactly reaches all n guiding positions the value of the approximation quality measure is $\odot = 100$. This is the maximum possible value for $\odot$. Planar dyads that move the workpiece near the n guiding positions will have values of $\odot$ near 100 while those that do not approach the guiding positions well will have smaller values of $\odot$; with the minimum possible value being $-\infty$. Note that the approximation quality measure $\odot$ is expressed in units of length; the length units used in defining the fixed axes.

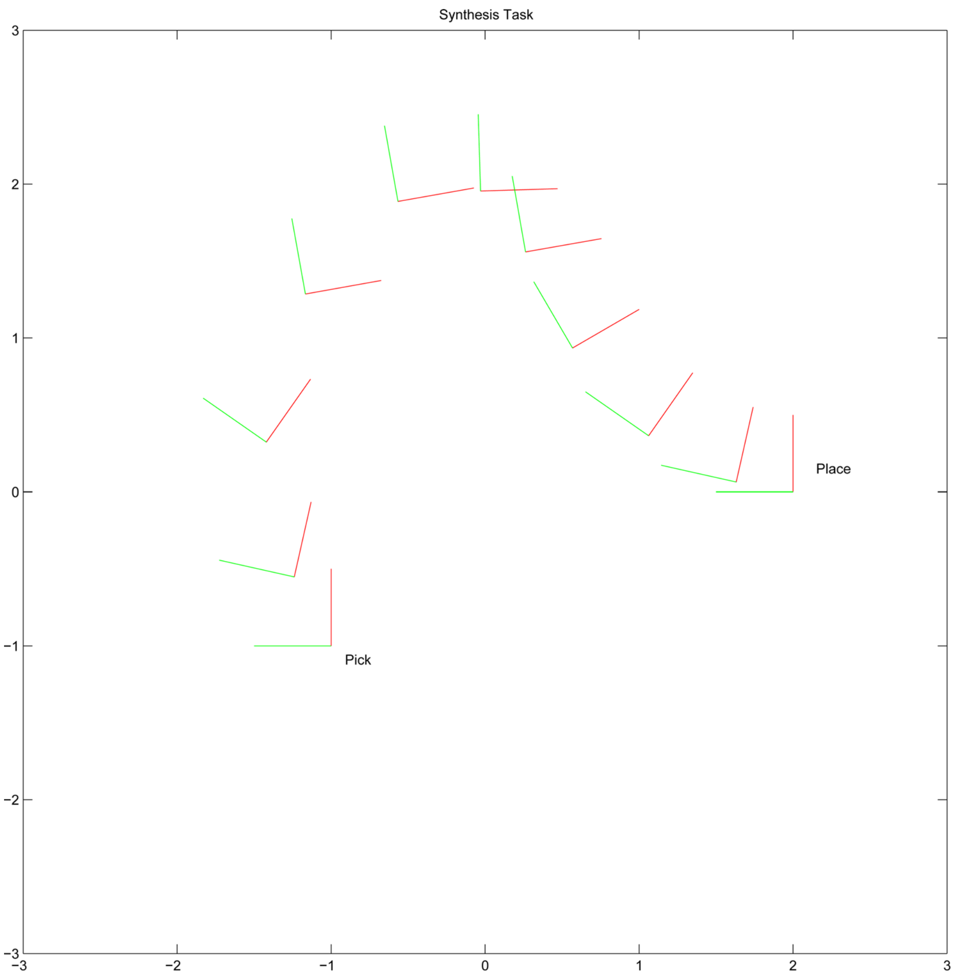

Fig. 9 Eleven prescribed positions

Table 2 Eleven prescribed positions

#	x	y	θ (deg)	Motion type
1	−1.0000	−1.0000	90.0000	Exact
2	−1.2390	−0.5529	77.3621	Approximate
3	−1.4204	0.3232	55.0347	Approximate
4	−1.1668	1.2858	30.1974	Approximate
5	−0.5657	1.8871	10.0210	Approximate
6	−0.0292	1.9547	1.7120	Approximate
7	0.2632	1.5598	10.0300	Approximate
8	0.5679	0.9339	30.1974	Approximate
9	1.0621	0.3645	55.0346	Approximate
10	1.6311	0.0632	77.3620	Approximate
11	2.0000	0.0000	90.0000	Exact

Kinematic Inversion

The above methodology may be reformulated to obtain the moving axis **v** for desired choice of fixed axis **u** by employing kinematic inversion. Inverting the relationship between the moving and fixed references frames yields the following analytical representation for the geometric constraint for a planar RR dyad [1,2]:

$$([A]^{-1}(\mathbf{u}-\mathbf{d})-\mathbf{v})\cdot([A]^{-1}(\mathbf{u}-\mathbf{d})-\mathbf{v})=d^2 \tag{5}$$

where $[A]^{-1}=[A]^{\mathrm{T}}$. In the above derivations we may use Eq. (5) in place of Eq. (1) to obtain the moving axis **v** for a desired fixed axis **u**.

Example #1

Here we employ the preceding methodology and design a planar 4R mechanism for five positions; two exact (the starting pick position and the final place position) and three guiding positions as shown in Fig. 4 and defined in Table 1 where $[A]=[\mathrm{Rot}_z(\theta)]$. In order to prescribe the size of the coupler link and to define the attachment of the moving body to the coupler these moving axes were selected: $\mathbf{v}_a=[0-1]^{\mathrm{T}}$ and $\mathbf{v}_b=[1-1]^{\mathrm{T}}$. The mixed synthesis algorithm yielded fixed axes: $\mathbf{u}_a=[9.2662,-4.2662]^{\mathrm{T}}$ and $\mathbf{u}_b=[12.3650,-3.8321]^{\mathrm{T}}$. The approximation quality measure for the dyads are $\odot=58.95$ and $\odot=53.71$, respectively. The perpendicular bisectors and three position solutions that illustrate the application of the algorithm to determine $\mathbf{u}_a$ are shown in Figs. 2 and 3 and those associated with $\mathbf{u}_b$ are shown in Fig. 5. The resulting closed chain is a Grashof double-rocker planar four-bar mechanism that does not suffer from circuit, branch, or order defects. Its link lengths are: input = 9.8249, coupler = 1.0000, output = 11.7125, and fixed = 3.1291. The solution mechanism is shown in Figs. 6–8 with green (light grey) input, yellow (medium grey) coupler, and red (dark grey) output links.

Example #2

We now employ the preceding methodology and design a planar 4R mechanism for 11 positions; the pick and place positions along with nine guiding positions as shown in Fig. 9 and defined in Table 2 where $[A]=[\mathrm{Rot}_z(\theta)]$. This rigid-body guidance problem was proposed by Michael McCarthy and Irvine [16]. In order to illustrate the inversion of the synthesis algorithm, desired fixed pivots were selected, and the algorithm was used to determine the

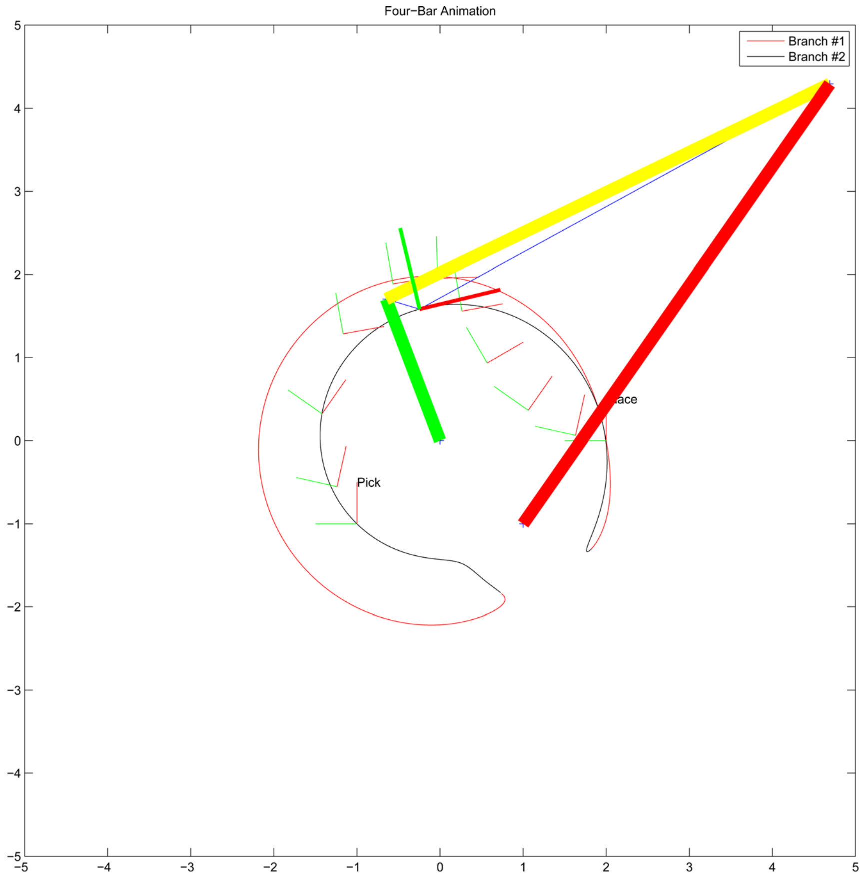

Fig. 10 The solution mechanism

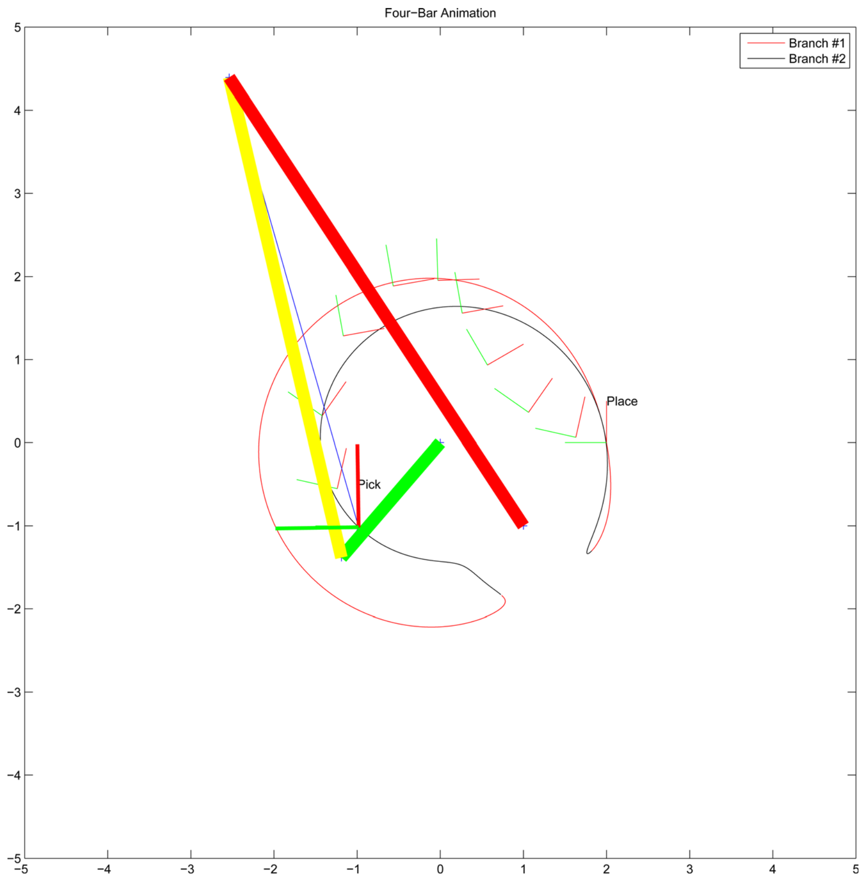

Fig. 11 The mechanism shown in the pick position

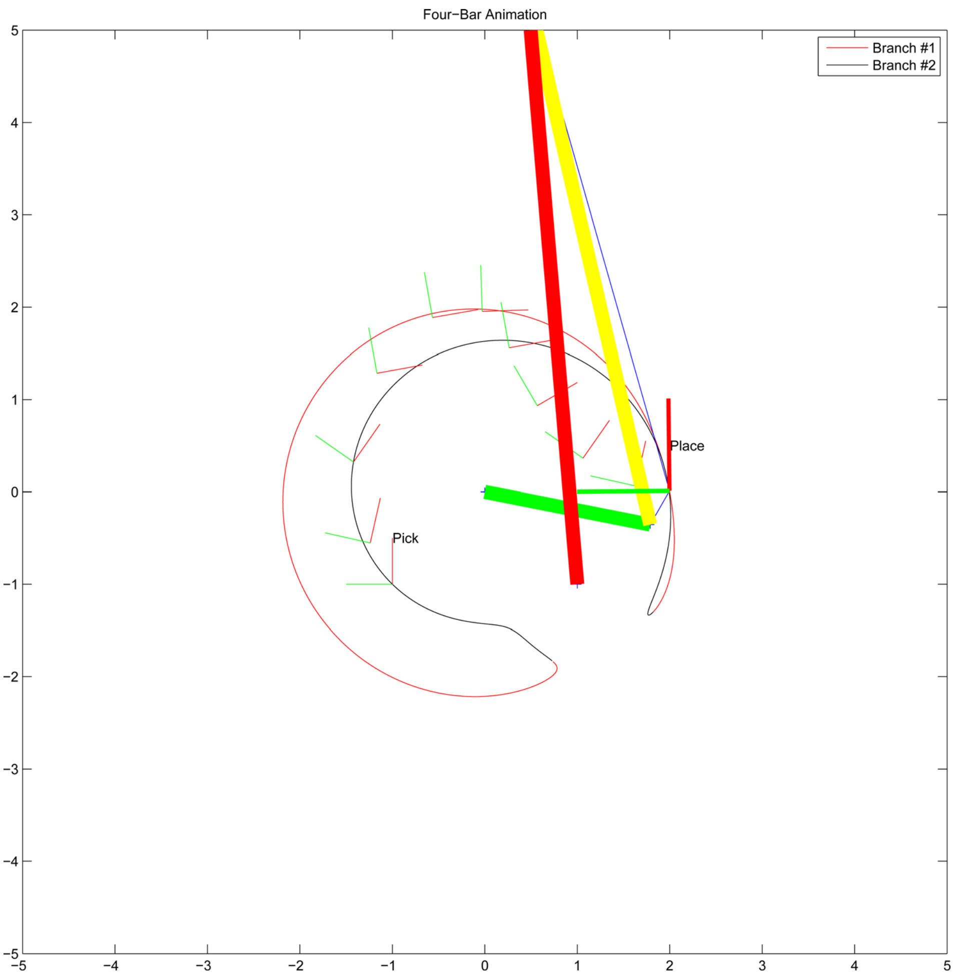

Fig. 12 The mechanism shown in the place position

corresponding moving pivots. Nine candidate fixed pivots were chosen by discretizing the desired fixed pivot area near the origin of the fixed frame; $[-1\ -1]^T$, $[-1\ 0]^T$, $[-1\ 1]^T$, $[0\ -1]^T$, $[0\ 0]^T$, $[0\ 1]^T$, $[1\ -1]^T$, $[1\ 0]^T$, and [11]^T. The algorithm was run using MATLAB 2012 on a standard WindowsXP laptop for each of the nine preceding candidate fixed pivots. The average run time was 0.497 s. All possible combinations of the nine candidate dyads yielded 36 candidate four-bar mechanisms. These 36 candidate solutions were analyzed by using a kinematic simulation in MATLAB [17]. Criteria for evaluating the 36 candidate mechanisms included motion near the guiding positions, location and orientation, and all mechanisms with circuit, branch, or order defects well rejected. The total design time, including running the algorithm nine times and evaluating the 36 candidate mechanisms, was less than 15 min. The final fixed axes were selected as: $\mathbf{u}_a = [1–1]^T$ and $\mathbf{u}_b = [0\ 0]^T$. The mixed synthesis algorithm yielded moving axes: $\mathbf{v}_a = [5.4329\ \ 1.4776]^T$ and $\mathbf{v}_b = [-0.3630\ \ 0.2123]^T$. The approximation quality measure for the dyads are $\odot = 14.577$ and $\odot = 81.64$, respectively. The resulting closed chain is a non-Grashof $\pi - \pi$ double-rocker planar four-bar mechanism that does not suffer from circuit, branch, or order defects. Its link lengths are: input = 1.82417, coupler = 5.9324, output = 6.4506, and fixed = 1.4142. The solution mechanism is shown in Figs. 10–12 with green input, yellow coupler, and red output links. Note in Fig. 10 how the moving body has nearly accomplished the ~90 (deg) rotation with respect to the pick and place positions as prescribed by the intermediate guiding positions.

Conclusions

A novel dimensional synthesis technique for solving the mixed exact and approximate motion synthesis problem for planar RR dyads and four-bar closed kinematic chains has been presented. Moreover, an approximation quality measure that indicates the degree to which the n approximate positions have been achieved has been proposed. The methodology uses an analytic representation of the planar RR dyad's rigid body constraint equation in combination with classical geometric constructions for exact motion synthesis to yield designs that exactly reach two prescribed positions while approximating n additional guiding positions.

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