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SYNTHESIS OF WATT II MECHANISMS FOR FOUR SIMULTANEOUS POSITIONS

Pierre Larochelle[*]
Robotics & Spatial Systems Lab
Dept. of Mechanical & Aerospace Eng.
Florida Institute of Technology
Melbourne, Florida 32901
Email: vvenkata@my.fit.edu

Jugesh Sundram
Robotics & Spatial Systems Lab
Dept. of Mechanical & Aerospace Eng.
Florida Institute of Technology
Melbourne, Florida 32901
Email: jsundram2012@my.fit.edu

Ronald A. Zimmerman II
Magna Seating
Novi, Michigan
Email: ron.zimmerman@magna.com

ABSTRACT

This article presents the kinematic synthesis of Watt II six-bar mechanisms for simultaneously guiding two bodies through four prescribed positions. The two bodies to be moved are connected by a revolute joint and the motion generation task is defined by the four desired positions of one body and the relative angle of the second body with respect to the first body. The methodology uses an algebraic geometry formulation of the exact synthesis of planar RR dyads for four prescribed positions from classical Burmester theory. The result is a dimensional synthesis technique for designing Watt II mechanisms for four simultaneous positions. A case study illustrating the application of the synthesis algorithm is included.

INTRODUCTION

This paper presents the kinematic dimensional synthesis of planar one degree of freedom Watt II six-bar mechanisms for simultaneously guiding two bodies through four finite positions. The two bodies to be moved are connected by a revolute joint. The motion generation task is defined by the four desired positions of one body and the relative angle of the second body with respect to the first body. Recent efforts to solve novel mechanism and linkage design problems have revealed that such simultaneous motion generation problems have practical application therefore having sound kinematic synthesis theory and methods to address

this new class of motion generation problem is desirable. This effort seeks to make such a contribution.

Dimensional synthesis is the determination of the necessary geometric dimensions of the links in a kinematic chain to create a mechanism that will effect a desired motion transformation [1]. Various graphical and analytical dimensional synthesis methods have been discussed by McCarthy and Soh [2], Erdman and Sandor [3], and Suh and Radcliffe [4]. In related works Mirth presented an algorithm for the design of planar four-bar mechanisms based upon synthesis algorithms that yield solutions that guide a body through four positions. The synthesis of planar RR dyads for four exact positions is a well known problem whose solution involves the center point and circle point curves of classical Burmester theory [1, 5, 6]. Mirth exploited the known properties of these curves to yield algorithms for two exact [7, 8] and three exact [9] positions and n approximate positions. Mirth solved multiple four position problems to yield multiple center or circle point curves. Graphically he identified regions of acceptable solutions from these curves. Holte et al. extended that work with a mathematical method for identifying the regions of acceptable solutions [10]. In [11] the method was extended to include approximate velocity constraints. Zhao, Schmiedeler, and Murray [12] have developed synthesis techniques for planar rigid body mechanisms to solve shape changing tasks. Their shape changing tasks is related to the simultaneous motion generation task considered here although the two synthesis techniques are significantly different. Using the preceding methods, the dimensional synthesis of mechanisms may be performed. Subsequently,

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[*]Address all correspondence to this author.

the dimensional synthesis parameters may be used to define the geometric parameters of the mechanism to produce a parametric model in computer-aided design software tools such as Creo Parameteric [13], Solidworks [14], and CATIA [15]. There have also been research efforts directed to producing dedicated software tools for the synthesis of spherical, planar and spatial mechanisms to solve motion generation problems. These include Lincages [16], Osiris [17,18], SPHINX [19], Spades [20], SPHINXPC [21], SPASUR [22]. Bourelle et al. [23] introduced a graphical user interface that solves the five-pose problem based on an algorithm described in [24]. Ch Mechanism Toolkit [25] is another notable software that has been used for the analysis and simulation of planar linkages.

This paper proceeds as follows. First, the specification of the motion generation task is discussed. Next, the kinematic dimensional synthesis method is presented. This is followed by an overview of a MATLAB [26] software tool that has been been developed to implement the method. Next, a case study illustrating the application of the synthesis algorithm to design Watt II six-bar mechanisms for simultaneously guiding two bodies connected by a revolute joint through four finite positions. Finally, planned future work, acknowledgements, and conclusions are presented.
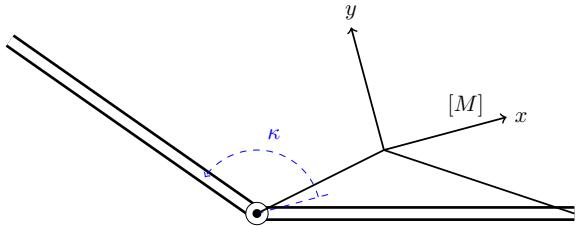
SYNTHESIS ALGORITHM

First, the motion generation task is discussed. Next, the synthesis algorithm is presented along with a *Step by Step* procedure for its implementation.

Motion Generation Task

The task at hand is to simultaneously generate the motion of two rigid bodies that are connected to each other by a revolute joint. This task is illustrated in Figure 1 in which a coordinate frame $[M]$ is attached to moving body #1 and moving body #2 is shown at an angle κ with respect to moving body #1. The motion generation task can be prescribed by the four parameters x, y, θ, and κ that are grouped into a quadruple; (x,y,θ,κ). Three of these parameters, x, y, and θ, define the position of the moving frame $[M]$ with respect to a fixed reference frame while κ defines the angle of moving body #2 with respect to the x-axis of $[M]$; i.e. moving body #1. Here, we are addressing simultaneous motion generation for four positions that will be prescribed by four quadruples $(x,y,\theta,\kappa)_i$ where $i = 1,\ldots,4$.

Synthesis Overview

The synthesis algorithm presented here for the Watt II six-bar mechanism consists of two phases. In the first phase a planar four-bar mechanism (Figure 2) is synthesized to guide moving body #1 through its four prescribed positions. In the second phase, an RR dyad is added to the four-bar to guide moving body #2,

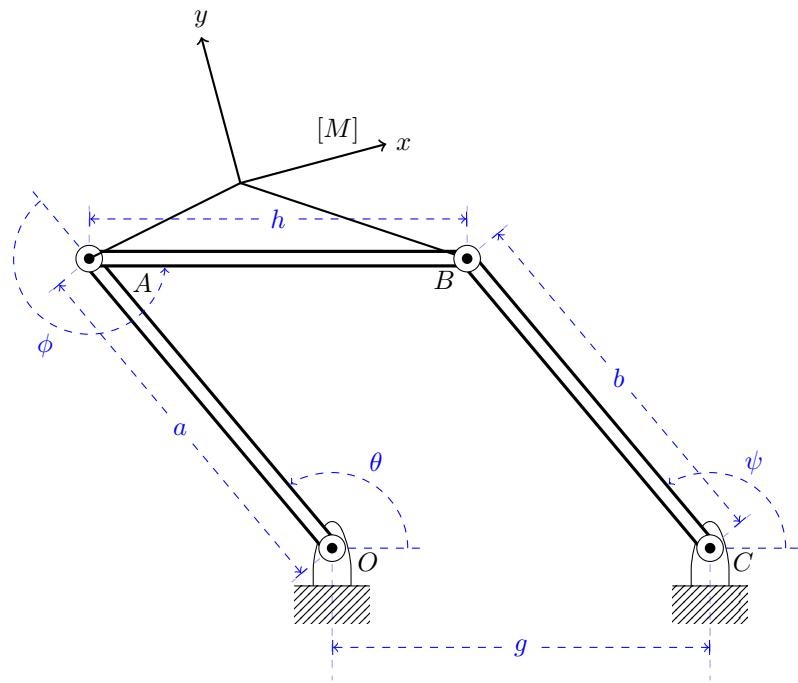**FIGURE 1**. **SIMULTANEOUS MOTION GENERATION: TASK SPECIFICATION**

connected to the body #1 by a revolute joint, at the prescribed angle in each of the four positions. The result is a Watt II six-bar mechanism as shown in Figure 3.

Both phase 1 and phase 2 of the synthesis algorithm require solving the dimensional synthesis of planar RR dyads for four prescribed positions. This is a well known motion generation problem and numerous solution methods are available. We chose to utilize classical Burmester theory methods as presented in [1, 5, 6]. Previously, we have developed MATLAB software tool that implements these methods along with an efficient and effective graphical user interface, see [27]. We utilize this MATLAB software to solve the planar RR dyad four position problem. The MATLAB software tool solves the four position problem by computing the corresponding center-point curve and circle-point curve; collectively known as the Burmester curves. The center-point curve is the locus of the fixed pivots of solution RR dyads and the circle-point curve is the locus of solution moving pivots. Thus, the points on the center-point and circle-point curves have a one-to-one onto correspondence with respect to each other. An RR dyad that moves a body through the four positions has fixed and moving pivots located on the Burmester curves. Figure 4 shows the graphical user interface of the planar RR dyad four position MATLAB. The interface consists of a Design Panel for prescribing the motion generation task and a Solution Space for presenting and interacting with the Burmester curves. Once the parameters defining the four positions have been entered into the Design Panel the Burmester curves are generated and the user selects a point on the curves displayed in the Solution Space to yield a solution planar RR dayd.
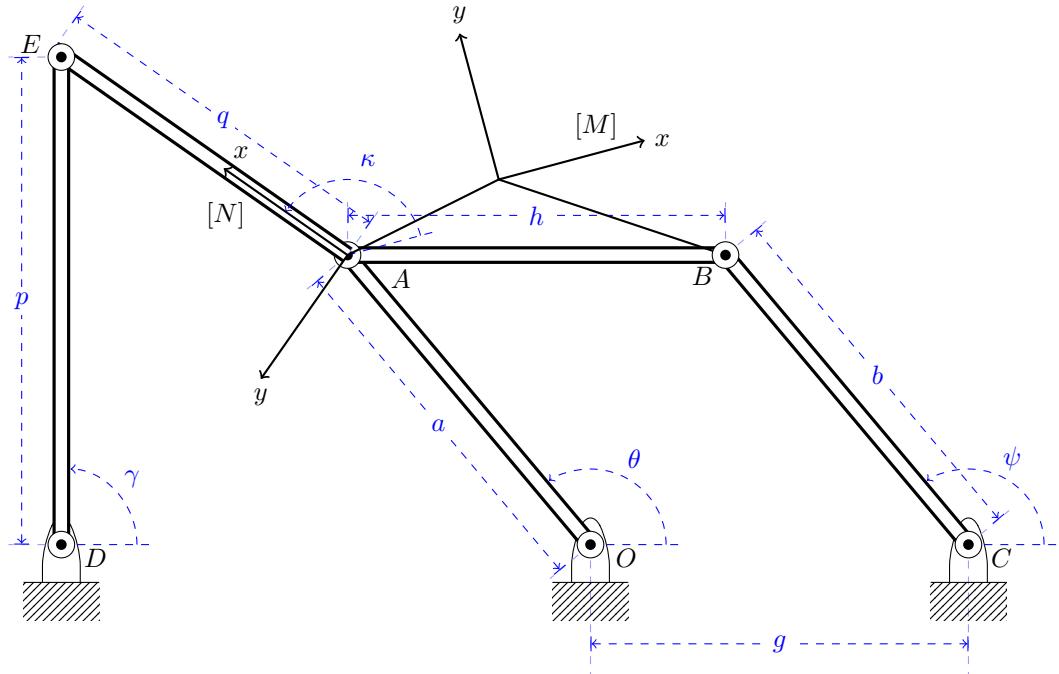
Step-by-Step: Synthesis Procedure

Here we present a *Step-by-Step* procedural implementation of the synthesis algorithm.

Problem Statement: Synthesize a planar Watt II mechanism to exactly guide two bodies simultaneously through

FIGURE 2. FOUR-BAR MECHANISM

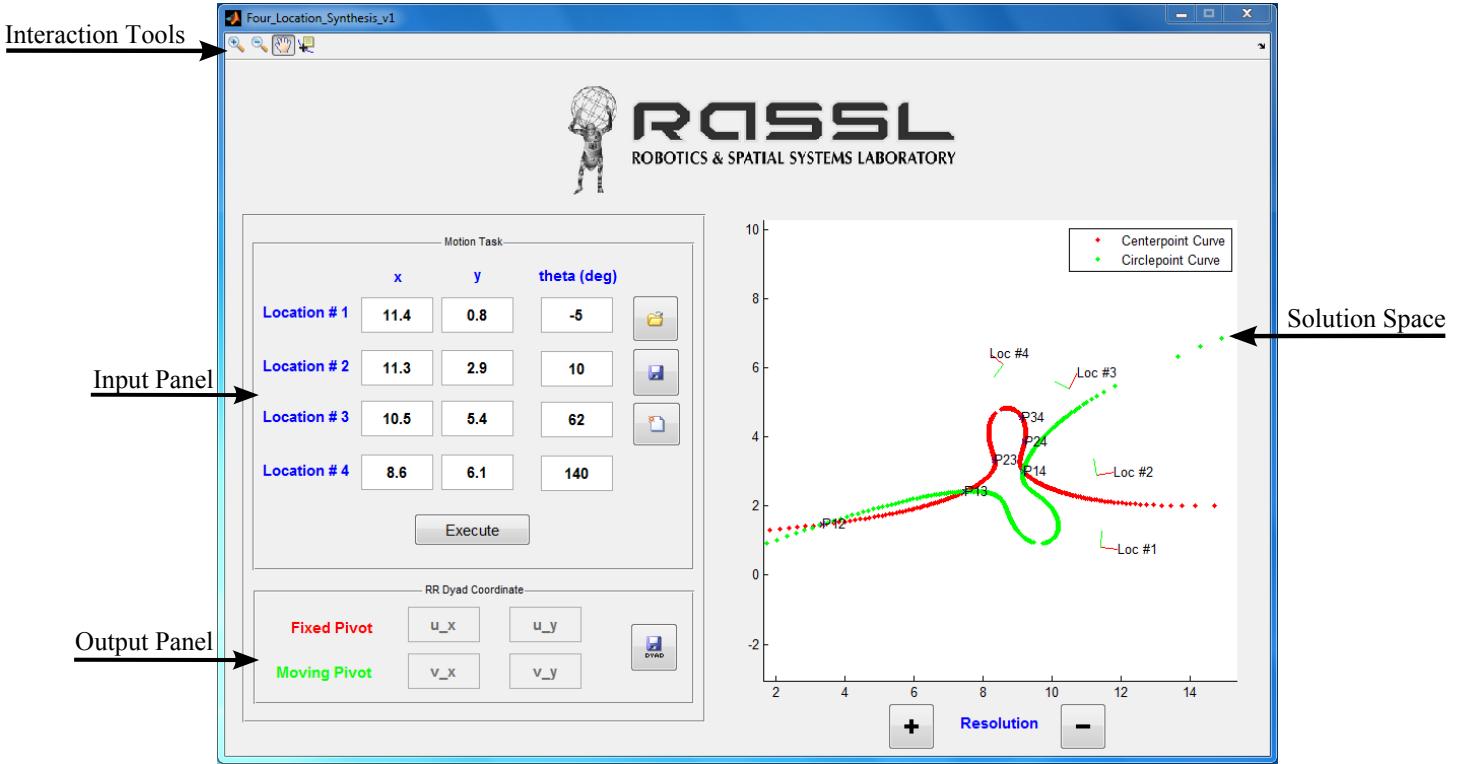**FIGURE 3.** WATT II SIX-BAR MECHANISM

3

FIGURE 4.   GUI FOR FOUR POSITION RR DYAD SYNTHESIS

four positions.

Given: Four exact positions for body #1 $[M]_i$ and four angles κ_i that orient body #2 with respect to body #1 in each of the four positions, $i = 1, \ldots, 4$.

Find: The planar Watt II six-bar mechanism that simultaneously guides body #1 & body #2 through their four prescribed positions.

1. *Phase 1*: Synthesize a planar four-bar mechanism $O - A - B - C$ that guides body #1 through the four prescribed positions $[M]_i$, see Figure 2. This is done by utilizing the MATLAB planar RR dyad synthesis software tool twice; once for the $\overline{OA}$ dyad and again for the $\overline{CB}$ dyad.
2. Determine the four prescribed positions of body #2, i.e. $[N]_i$, using the fact that body #1 is connected to body #2 at its moving pivot A, see Figures 2 and 3. Frame $[N]_i$ is defined by having its origin at A_i and being oriented at an angle κ_i with respect to $[M]_i$; $[N]_i = [M]_i[Q][Rot_z(\kappa_i)]$ where $[Q]$ is the planar homogeneous transform representation of the pure translation from the origin of $[M]$ to the moving pivot A.

3. *Phase 2*: Synthesize a planar RR dyad $\overline{DE}$ that guides body #2 through the four prescribed positions $[N]_i$. Again, this is done by utilizing the MATLAB planar RR dyad synthesis software tool.
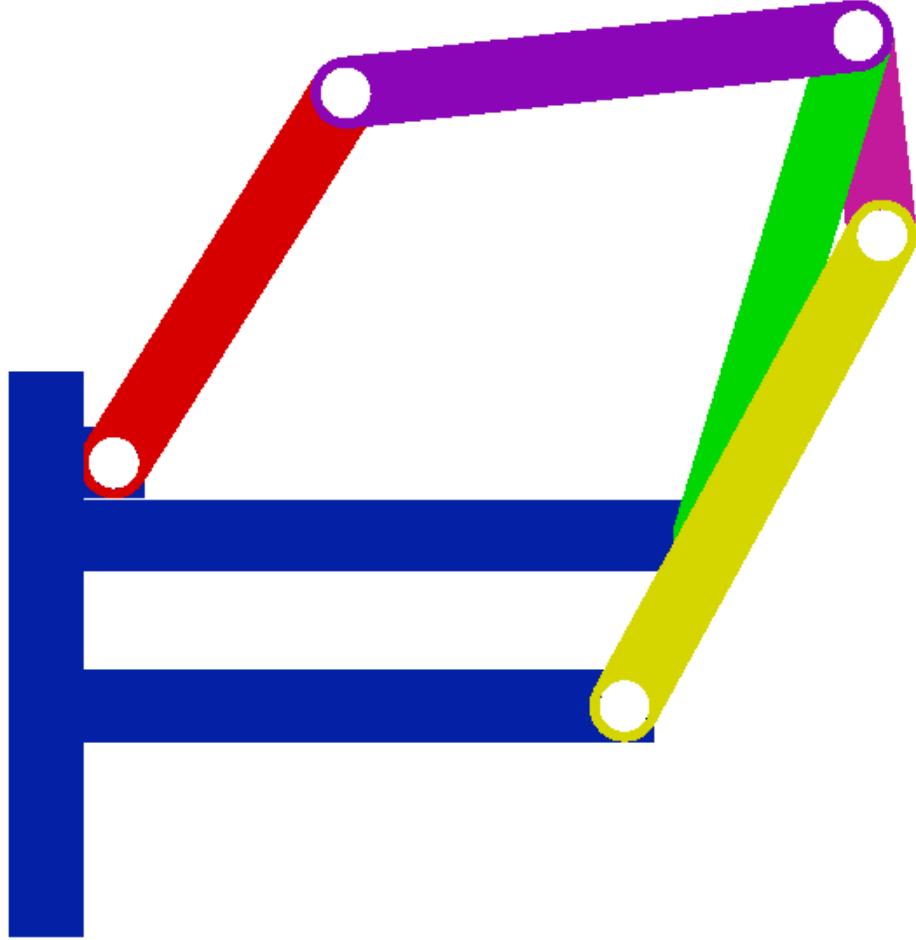
4. Add this dyad to the four-bar $O - A - B - C$ to yield a Watt II planar six-bar mechanism as shown in Figure 3.

CASE STUDY

We now proceed to design a Watt II six-bar mechanism for the 4 simultaneous position quadruples found in Table 1. Recall that the four positions of body #1 are prescribed by the triple (x, y, θ) and the relative angle of body #2 with respect to body #1 in each position is given by κ.

The *Step-by-Step* synthesis procedure and related MATLAB software were utilized to determine a viable solution to this simultaneous motion generation problem. Phase 1 produced the following four-bar mechanism that guides body #1 as desired. Dyad $\overline{OA}$ has fixed pivot $O = [4.4613 \ -2.2148]^T$ and moving pivot $A = [0.4935 \ -0.1385]^T$. Dyad $\overline{CB}$ has fixed pivot $C = [4.3906 \ -2.8053]^T$ and moving pivot $B = [0.5479 \ -3.9986]^T$. Subsequently, the four locations of moving body #2, i.e. frame $[N]$, were computed and are listed in Table 2. Phase 2 yielded the following dyad to complete the Watt II six-bar mechanism. Dyad $\overline{DE}$ has fixed pivot $D = [4.2249 \ -2.7222]^T$ and moving pivot $E = [0.4030 \ -3.8670]^T$.
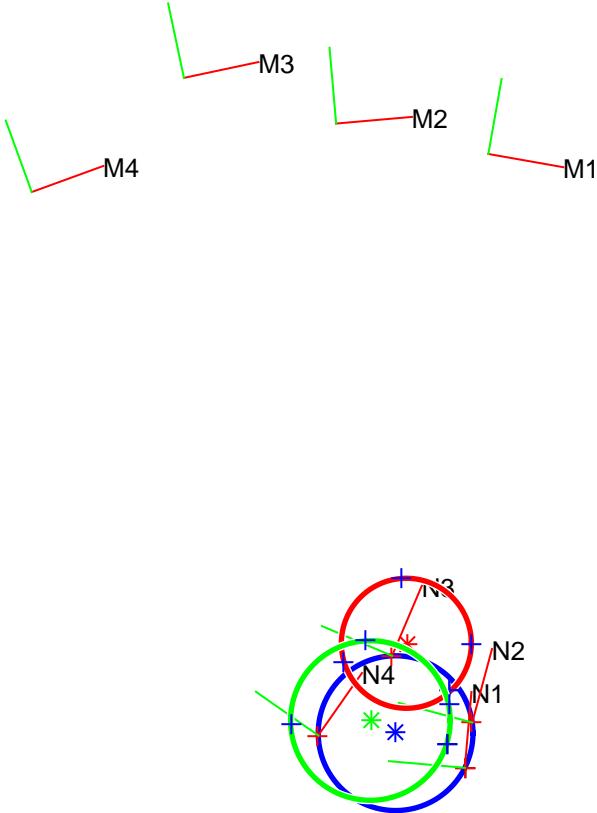
FIGURE 5. THE SOLUTION MECHANISM

The three solutions dyads were then combined, according to Figure 3, to yield a one degree of freedom Watt II six-bar mechanism that solves the prescribed simultaneous motion generation task. This Watt II six-bar mechanism is shown in Figure 5. A verification study of the solution mechanism was conducted and the results are shown in Figure 6. In this figure the four positions of body #1 ($[M]_i$) and body #2 ($[N]_i$) are shown. The locations of the moving pivots in each of their four positions were determined and plotted along with their corresponding centers (*), or fixed pivots. Observe that these moving pivots travel on circles centered about their corresponding fixed pivots. This verifies that these RR dyads do indeed guide their corresponding moving bodies through the four simultaneous positions.

TABLE 1. FOUR PRESCRIBED SIMULTANEOUS POSITIONS

Pos. #	x	y	θ(deg)	κ(deg)
1	5.00	1.00	-10.00	95.0
2	4.00	1.20	5.00	70.0
3	3.00	1.50	12.00	55.0
4	2.00	0.75	20.00	35.0

FUTURE WORK

Current efforts are focused on modifying the synthesis method presented here to address the design of Stephenson III six-

FIGURE 6. **VERIFYING THE SOLUTION MECHANISM**

TABLE 2. **FOUR COMPUTED POSITIONS OF BODY #2**

Pos. #	x	y	θ (deg)
1	4.8452	-3.0330	85.00
2	4.8943	-2.7356	75.00
3	4.3673	-2.2973	67.00
4	3.8824	-2.8201	55.00

bar mechanisms for simultaneously guiding two bodies,connected by a revolute joint, through four finite positions.

CONCLUSION

This paper presents a novel method for synthesizing Watt II six-bar mechanisms for four simultaneous positions. This dimensional synthesis algorithm is based upon classical Burmester theory for designing planar RR dyads for generating motion through four positions. A case study illustrating the application of the synthesis technique was included. Efforts are underway to extend the ideas presented here to yield an algorithm for the synthesis of Stephenson III six-bar mechanisms for four simultaneous positions.

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